Goéland: A Concurrent Tableau-Based Theorem Prover

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Context

Method of analytic tableaux

- Free variables
- Usually managed sequentially

Fair proof search is difficult!

- Shared free variables
- Find a subtitution for the whole tree
- Completeness issues: branch selection, free variables reintroduction

Motivating example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

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Motivating example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$\frac{P(a), \neg P(b), \forall \boldsymbol{x}\ \boldsymbol{(P(x) \Leftrightarrow (\forall y\ P(y)))}}{P(X) \Leftrightarrow (\forall y\ P(y))}\ \gamma\forall$$

Motivating example

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$$\dfrac{\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{\boldsymbol{P(X) \Leftrightarrow (\forall y\ P(y))}}\ \gamma_\forall}{P(X), \forall y\ P(y) \qquad \neg P(X), \neg(\forall y\ P(y))}\ \beta_\Leftrightarrow$$

Motivating example

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Motivating example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$\cfrac{\cfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P({\color{red}a}) \Leftrightarrow (\forall y\ P(y))}\ \gamma_\forall}{\cfrac{P({\color{red}a}), \boldsymbol{\forall y\ P(y)}}{P(Y)}\ \gamma_\forall \qquad \cfrac{\neg P({\color{red}a}), \neg(\forall y\ P(y))}{\sigma = \{X \mapsto a\}}\ \odot_\sigma}\ \beta_\Leftrightarrow$$

Motivating example

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Motivating example (other branch)

$$P(a), \neg P(b), \forall x \ (P(x) \Leftrightarrow (\forall y \ P(y)))$$

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Motivating example (other branch)

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$\dfrac{\dfrac{P(a), \neg P(b), \boldsymbol{\forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}}{P(\color{red}{b}\color{black}) \Leftrightarrow (\forall y\ P(y))}\ \gamma_{\forall}}{\dfrac{P(\color{red}{b}\color{black}), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\ \odot_{\sigma} \qquad \dfrac{\dfrac{\neg P(\color{red}{b}\color{black}), \neg(\forall y\ P(y))}{\neg P(sko)}\ \delta_{\neg\forall}}{P(X_2) \Leftrightarrow (\forall y\ P(y))}\ reintroduction}\ \beta_{\Leftrightarrow}$$

Motivating example (other branch)

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$$\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(\color{red}{b}) \Leftrightarrow (\forall y\ P(y))}\ \gamma_{\forall}$$

$$\dfrac{P(\color{red}{b}) \Leftrightarrow (\forall y\ P(y))}{P(\color{red}{b}), \forall y\ P(y) \qquad \neg P(\color{red}{b}), \neg(\forall y\ P(y))}\ \beta_{\Leftrightarrow}$$

$$\dfrac{P(\color{red}{b}), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\ \odot_{\sigma}$$

$$\dfrac{\neg P(\color{red}{b}), \neg(\forall y\ P(y))}{\neg P(sko)}\ \delta_{\neg\forall}$$

$$\dfrac{\neg P(sko)}{\boldsymbol{P(X_2) \Leftrightarrow (\forall y\ P(y))}}\ reintroduction$$

$$\dfrac{\boldsymbol{P(X_2) \Leftrightarrow (\forall y\ P(y))}}{P(X_2), \forall y\ P(y) \qquad \neg P(X_2), \neg(\forall y\ P(y))}\ \beta_{\Leftrightarrow}$$

Motivating example (other branch)

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$$\dfrac{P({\color{red}b}) \Leftrightarrow (\forall y\ P(y))}{P({\color{red}b}), \forall y\ P(y) \qquad \neg P({\color{red}b}), \neg(\forall y\ P(y))}\ \beta_\Leftrightarrow$$

$$\dfrac{P({\color{red}b}), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\ \odot_\sigma$$

$$\dfrac{\neg P({\color{red}b}), \neg(\forall y\ P(y))}{\boldsymbol{\neg P(sko)}}\ \delta_{\neg\forall}$$

$$\dfrac{\boldsymbol{\neg P(sko)}}{P({\color{red}b}) \Leftrightarrow (\forall y\ P(y))}\ reintroduction$$

$$\dfrac{P({\color{red}b}) \Leftrightarrow (\forall y\ P(y))}{\boldsymbol{P({\color{red}b})}, \forall y\ P(y) \qquad \neg P({\color{red}b}), \neg(\forall y\ P(y))}\ \beta_\Leftrightarrow$$

$$\dfrac{\boldsymbol{P({\color{red}b})}, \forall y\ P(y)}{\boldsymbol{\sigma = \{X_2 \mapsto b\}}}\ \odot_\sigma$$

$$\boldsymbol{\sigma' = \{X_2 \mapsto sko\}}$$

Motivating example (other branch)

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$
\dfrac{\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(\color{red}{b}\color{black}) \Leftrightarrow (\forall y\ P(y))}\ \gamma_{\forall}}
{\dfrac{P(\color{red}{b}\color{black}), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\ \odot_\sigma
\qquad
\dfrac{\dfrac{\dfrac{\neg P(\color{red}{b}\color{black}), \neg(\forall y\ P(y))}{\neg P(sko)}\ \delta_{\neg\forall}}{P(\color{red}{b}\color{black}) \Leftrightarrow (\forall y\ P(y))}\ reintroduction}
{\dfrac{P(\color{red}{b}\color{black}), \forall y\ P(y)}{\sigma = \{X_2 \mapsto b\}}\ \odot_\sigma
\qquad
\dfrac{\neg P(\color{red}{b}\color{black}), \boldsymbol{\neg(\forall y\ P(y))}}{\neg P(sko_2)}\ \delta_{\neg\forall}}\ \beta_{\Leftrightarrow}}\ \beta_{\Leftrightarrow}
$$

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$$\dfrac{\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(b) \Leftrightarrow (\forall y\ P(y))}\,\gamma_{\forall}}{\dfrac{P(b), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\,\odot_\sigma \qquad \dfrac{\dfrac{\dfrac{\neg P(b), \neg(\forall y\ P(y))}{\neg P(sko)}\,\delta_{\neg\forall}}{P(b) \Leftrightarrow (\forall y\ P(y))}\,reintroduction}{\dfrac{P(b), \forall y\ P(y)}{\begin{array}{c}\sigma = \{X_2 \mapsto b\} \\ \sigma' = \{X_2 \mapsto sko\}\end{array}}\,\odot_\sigma \qquad \dfrac{\dfrac{\neg P(b), \neg(\forall y\ P(y))}{\neg P(sko_2)}\,\delta_{\neg\forall}}{\ldots}\,reintroduction}\,\beta_{\Leftrightarrow}}\,\beta_{\Leftrightarrow}$$

Exploring branches in parallel?

Approach

- Each branch searches for a local solution
- Manages multiple solutions
- No more branch selection fairness problem

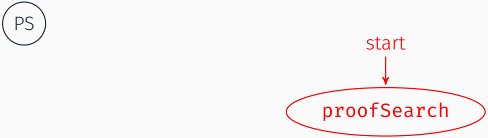
New challenges

- Free variable dependency
- Communication between branches

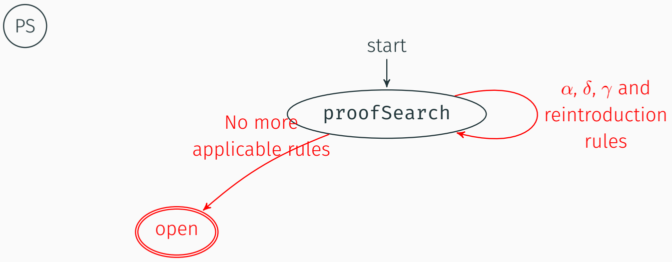
Technical point

- Backtracking on solutions
- Reintroduction fairness problem: iterative deepening

Procedures interactions



Procedures interactions



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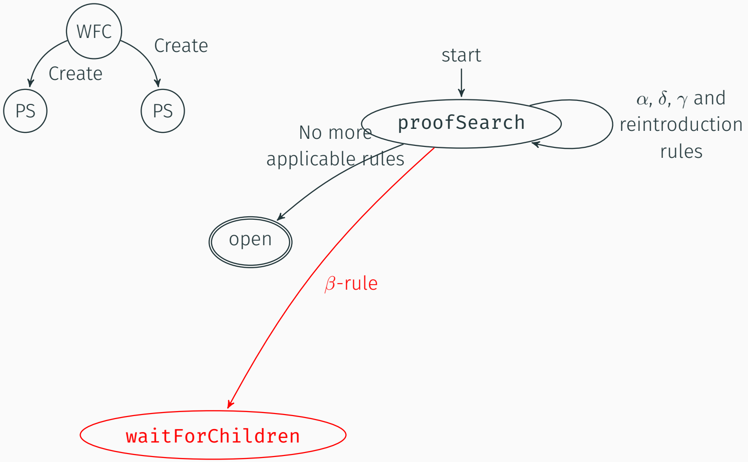

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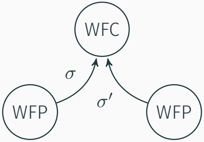

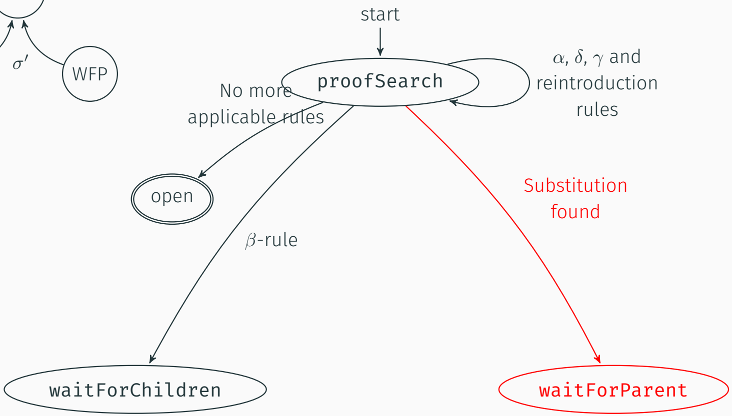

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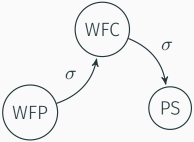

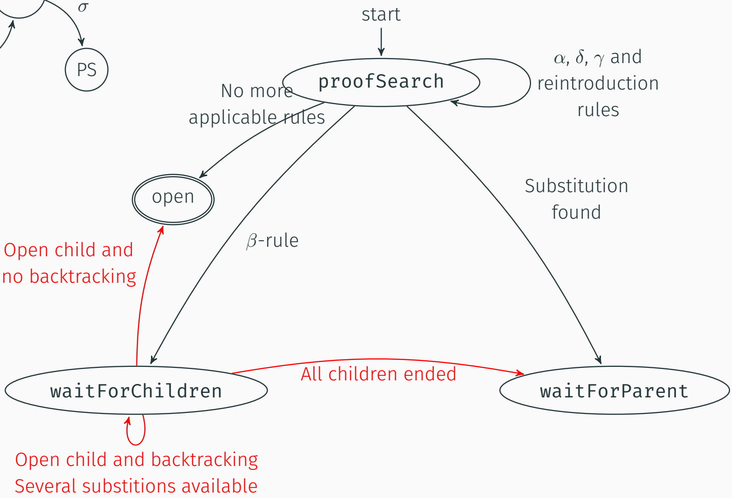

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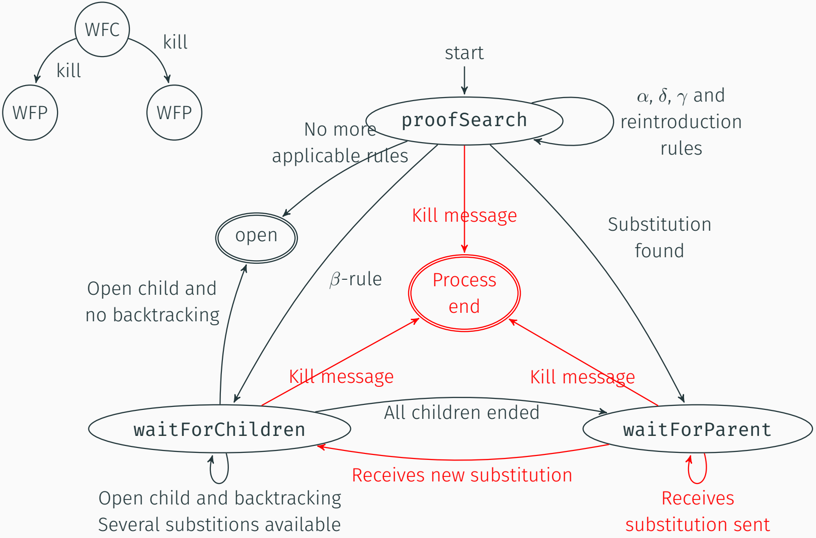

Come back to example

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$\sigma = \{X \mapsto b\}$ $\sigma = \{X \mapsto a\}$

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Closed

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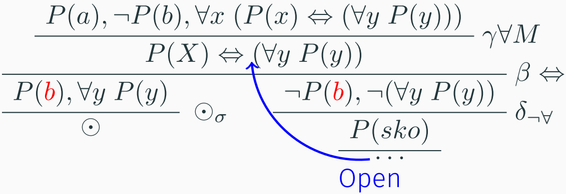

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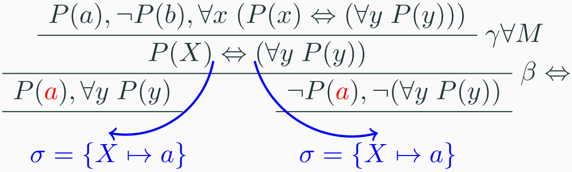

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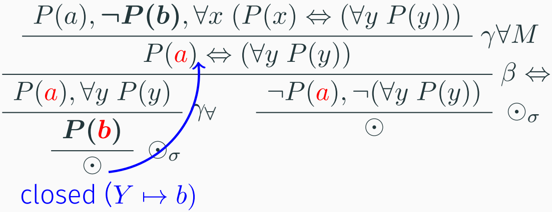
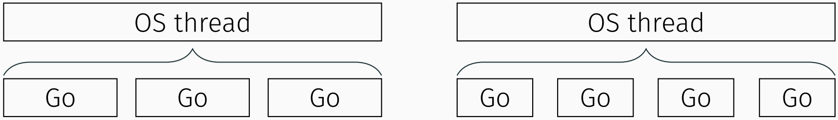


Goéland tool

Implementation

- Go programming language
- Designed for concurrency
- Goroutines: N:M lightweight threads



Experimentals results on TPTP

	SYN (263 problems)		SET (464 problems)	
Goéland	199		229	
Zenon	256	$(+60, -3)$	150	$(+74, -153)$
Princess	195	$(+1, -5)$	258	$(+132, -103)$
LeoIII	195	$(+1, -5)$	177	$(+93, -145)$
E	261	$(+62, -0)$	363	$(+184, -50)$
Vampire	262	$(+63, -0)$	321	$(+167, -75)$

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Reasoning Modulo Theory

Example

- Axiom: $\forall a, b.\ a \subseteq b \Leftrightarrow \forall x.\ x \in a \Rightarrow x \in b$
- Axiom: $\forall a, b.\ a = b \Leftrightarrow a \subseteq b \wedge b \subseteq a$
- Conjecture: $\forall a.\ a \subseteq a$

In the method of analytics tableaux

$(\forall a, b.\ a \subseteq b \Leftrightarrow \forall x.\ x \in a \Rightarrow x \in b) \wedge (\forall a, b.\ a = b \Leftrightarrow a \subseteq b \wedge b \subseteq a) \wedge \neg(\forall a.\ a \subseteq a)$

Reasoning Modulo Theory

$$\dfrac{(\forall a, b.\ a \subseteq b \Leftrightarrow \forall x.\ x \in a \Rightarrow x \in b) \wedge (\forall a, b.\ a = b \Leftrightarrow a \subseteq b \wedge b \subseteq a) \wedge \neg(\forall a.\ a \subseteq a)}{\forall a, b.\ a \subseteq b \Leftrightarrow \forall x.\ x \in a \Rightarrow x \in b, \forall a, b.\ a = b \Leftrightarrow a \subseteq b \wedge b \subseteq a, \neg(\forall a.\ a \subseteq a)}\ \alpha_\wedge$$

$$\dfrac{\dfrac{\dfrac{\cdots}{\neg(a \subseteq a)}\ \delta_{\neg\forall}}{(\forall b.\ A \subseteq b \Leftrightarrow \forall x.\ x \in A \Rightarrow x \in b)}\ \gamma_{\forall M}}{(A \subseteq B \Leftrightarrow \forall x.\ x \in A \Rightarrow x \in B)}\ \gamma_{\forall M}$$

$$\dfrac{(A \subseteq B \Leftrightarrow \forall x.\ x \in A \Rightarrow x \in B)}{\dfrac{A \subseteq B, x \in A \Rightarrow x \in B}{\sigma = \{A \mapsto a, B \mapsto a\}}\ \odot_\sigma \qquad \dfrac{\neg(A \subseteq B), \neg(\forall x.\ x \in A \Rightarrow x \in B)}{\dfrac{\neg(a \subseteq a), \neg(\forall x.\ x \in a \Rightarrow x \in a)}{\dfrac{\neg(x \in a \Rightarrow x \in a)}{\dfrac{\neg(x \in a), (x \in a)}{\odot}\ \odot}\ \alpha_{\neg\Rightarrow}}\ \delta_{\neg\forall}}\ \sigma}\ \beta_\Leftrightarrow$$

Deduction Modulo Theory (DMT)

Main heuristic

$(\forall \vec{x}.)\ A \Leftrightarrow F$ where:

- A is an atomic formula
- F is a non-atomic formula

Axiom: $\forall a, b.\ a \subseteq b \Leftrightarrow \forall x.\ x \in a \Rightarrow x \in b$
Rule: $A \subseteq B \rightarrow \forall x.\ x \in A \Rightarrow x \in B$

Axiom: $\forall a, b.\ a = b \Leftrightarrow a \subseteq b \wedge b \subseteq a$
Rule: $A = B \rightarrow A \subseteq B \wedge B \subseteq A$

Deduction Modulo Theory (DMT)

Rewrite rules

$$A \subseteq B \rightarrow \forall x.\ x \in A \Rightarrow x \in B$$
$$A = B \rightarrow A \subseteq B \wedge B \subseteq A$$

$$\neg(\forall a.\ a \subseteq a)$$

Deduction Modulo Theory (DMT)

Rewrite rules

$$A \subseteq B \to \forall x.\ x \in A \Rightarrow x \in B$$
$$A = B \to A \subseteq B \land B \subseteq A$$

$$\frac{\neg(\forall a.\ a \subseteq a)}{\neg(a \subseteq a)}\ \delta_{\neg\forall}$$

Deduction Modulo Theory (DMT)

Rewrite rules

$$A \subseteq B \rightarrow \forall x.\ x \in A \Rightarrow x \in B$$
$$A = B \rightarrow A \subseteq B \wedge B \subseteq A$$

$$\frac{\neg(\forall a.\ a \subseteq a)}{\neg(a \subseteq a)}\ \delta_{\neg\forall}$$

Deduction Modulo Theory (DMT)

Rewrite rules

$$A \subseteq B \to \forall x.\ x \in A \Rightarrow x \in B$$
$$A = B \to A \subseteq B \land B \subseteq A$$

$$\dfrac{\dfrac{\neg(\forall a.\ a \subseteq a)}{\neg(a \subseteq a)}\,\delta_{\neg\forall}}{\neg(\forall x.\ x \in a \Rightarrow x \in a)} \to (A \mapsto a, B \mapsto a)$$

Deduction Modulo Theory (DMT)

Rewrite rules

$$A \subseteq B \rightarrow \forall x.\ x \in A \Rightarrow x \in B$$
$$A = B \rightarrow A \subseteq B \wedge B \subseteq A$$

$$\cfrac{\cfrac{\neg(\forall a.\ a \subseteq a)}{\neg(a \subseteq a)}\ \delta_{\neg\forall}}{\neg(\forall x.\ x \in a \Rightarrow x \in a)} \rightarrow (A \mapsto a, B \mapsto a)$$

Deduction Modulo Theory (DMT)

Rewrite rules

$$A \subseteq B \rightarrow \forall x.\ x \in A \Rightarrow x \in B$$
$$A = B \rightarrow A \subseteq B \wedge B \subseteq A$$

$$\dfrac{\dfrac{\dfrac{\neg(\forall a.\ a \subseteq a)}{\neg(a \subseteq a)}\ \delta_{\neg\forall}}{\neg(\forall x.\ x \in a \Rightarrow x \in a)} \rightarrow (A \mapsto a, B \mapsto a)}{\neg(x \in a \Rightarrow x \in a)}\ \delta_{\neg\forall}$$

Deduction Modulo Theory (DMT)

Rewrite rules

$$A \subseteq B \rightarrow \forall x.\ x \in A \Rightarrow x \in B$$
$$A = B \rightarrow A \subseteq B \wedge B \subseteq A$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\forall a.\ a \subseteq a)}{\neg(a \subseteq a)}\,\delta_{\neg\forall}}{\neg(\forall x.\ x \in a \Rightarrow x \in a)} \rightarrow (A \mapsto a, B \mapsto a)}{\neg(x \in a \Rightarrow x \in a)}\,\delta_{\neg\forall}}{\neg(x \in a), (x \in a)}\,\alpha_{\neg\Rightarrow}$$

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$$\begin{array}{ll}
\neg(\forall a.\ a \subseteq a) & \\
\hline
\neg(a \subseteq a) & \delta_{\neg\forall} \\
\hline
\neg(\forall x.\ x \in a \Rightarrow x \in a) & \rightarrow (A \mapsto a, B \mapsto a) \\
\hline
\neg(x \in a \Rightarrow x \in a) & \delta_{\neg\forall} \\
\hline
\neg(x \in a), (x \in a) & \alpha_{\neg\Rightarrow} \\
\hline
\odot & \odot
\end{array}$$

Deduction Modulo Theory (DMT)

Benefits

- Avoid combinatorial explosion
- “Useless” axioms aren’t tiggered
- Shorter proof
- Not limited to one theory
- Good properties for an ATP!

Deduction Modulo Theory (DMT)

They use it too!

- iProverModulo: search time divided by 10 in average (compared to iProver)
- ZenonModulo: from 48.5% to 80.3% on BWare benchmark (compared to Zenon)
- Zipperposition: use of deduction modulo theory on Tarski's geometry
- ArchSAT: dealing with static and dynamic rewrite systems

Experimentals results on TPTP

	SYN (263 problems)		SET (464 problems)	
GoélandDMT	199		272	
Goéland	199	$(+0, -0)$	229	$(+23, -66)$
Zenon	256	$(+60, -3)$	150	$(+57, -179)$
Princess	195	$(+1, -5)$	258	$(+104, -118)$
LeoIII	195	$(+1, -5)$	177	$(+73, -168)$
E	261	$(+62, -0)$	363	$(+153, -62)$
Vampire	262	$(+63, -0)$	321	$(+136, -87)$

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Analysis and future work

Analysis

- Fairness between branches managed by concurrency
- Efficiency of DMT to reason inside of a theory
- Promising results for a very new prover, especially with DMT

Future work

- Completeness proof
- Polymorphic types
- Arithmetic (with simplex and branch and bound)

Thank you!

`https://github.com/GoelandProver/Goeland`

Tableaux

$$\frac{\bot, \neg\top, P \neg Q}{\odot}\ \odot \qquad \frac{\alpha}{\begin{array}{c}\alpha_1\\ \alpha_2\end{array}}\ \alpha \qquad \frac{\beta}{\beta_1 \mid \beta_2}\ \beta$$

Where $\sigma(P) = \sigma(Q)$

$$\frac{(\exists/\neg\forall)x.\ \delta(x)}{\delta_1(x \leftarrow f(args))}\ \delta \qquad \frac{(\forall/\neg\exists)x.\ \gamma(x)}{\gamma_1(x \leftarrow X)}\ \gamma$$

Where f is a fresh skolem symbol and $args$ the free variables in δ

Where X is a new variable not occuring anywhere else and waiting for an instanciation

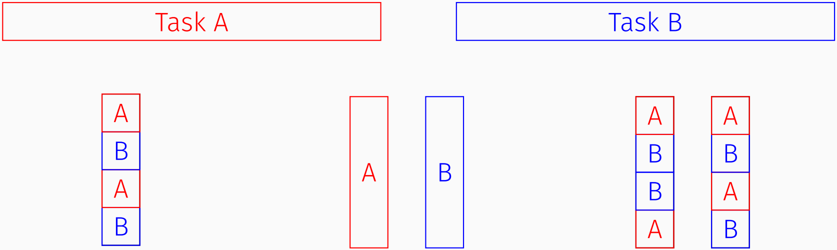
Concurrency vs. parallelism

Concurrency

Concurrency is about an application making progress on more than one task at the same time.

Parallelism

Parallelism is about tasks which can be processed in parallel, for instance on multiple CPUs at the exact same time.



Concurrent but not parallel

Parallel but not concurrent

parallel and concurrent