Goéland: A Concurrent Tableau-Based Theorem Prover (System Description)

IJCAR2022

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Context

Method of analytic tableaux

- Free variables
- Usually managed sequentially

Fair proof search is difficult!

- Shared free variables
- Find a subtitution for the whole tree
- Completeness issues: branch selection, free variables reintroduction

Motivating example

$$P(a), \neg P(b), \forall x \; (P(x) \Leftrightarrow (\forall y \; P(y)))$$

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Motivating example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$\frac{P(a), \neg P(b), \forall \boldsymbol{x}\ \boldsymbol{(P(x) \Leftrightarrow (\forall y\ P(y)))}}{P(X) \Leftrightarrow (\forall y\ P(y))}\ \gamma\forall$$

Motivating example

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$$\dfrac{\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{\boldsymbol{P(X) \Leftrightarrow (\forall y\ P(y))}}\ \gamma_\forall}{P(X), \forall y\ P(y) \qquad \neg P(X), \neg(\forall y\ P(y))}\ \beta_\Leftrightarrow$$

Motivating example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$\dfrac{\dfrac{\boldsymbol{P(a)}, \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(\color{red}{a}\color{black}) \Leftrightarrow (\forall y\ P(y))}\ \gamma_{\forall}}{P(\color{red}{a}\color{black}), \forall y\ P(y) \qquad \dfrac{\boldsymbol{\neg P(\color{red}{a}\color{black})}, \neg(\forall y\ P(y))}{\boldsymbol{\sigma = \{X \mapsto a\}}}\ \odot_{\sigma}}\ \beta_{\Leftrightarrow}$$

Motivating example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

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Motivating example

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Motivating example (other branch)

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

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$$\dfrac{\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P({\color{red}b}) \Leftrightarrow (\forall y\ P(y))}\ \gamma_{\forall}}{\dfrac{P({\color{red}b}), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\ \odot_{\sigma} \qquad \dfrac{\neg P({\color{red}b}), \neg(\forall y\ P(y))}{\dfrac{\neg P(sko)}{\dfrac{\boldsymbol{P(X_2) \Leftrightarrow (\forall y\ P(y))}}{P(X_2), \forall y\ P(y) \qquad \neg P(X_2), \neg(\forall y\ P(y))}\ \beta_{\Leftrightarrow}}\ reintroduction}\ \delta_{\neg\forall}}\ \beta_{\Leftrightarrow}$$

Motivating example (other branch)

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$$\dfrac{P(b) \Leftrightarrow (\forall y\ P(y))}{\dfrac{P(b), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\ \odot_{\sigma} \qquad \dfrac{\neg P(b), \neg(\forall y\ P(y))}{\dfrac{\boldsymbol{\neg P(sko)}}{P(b) \Leftrightarrow (\forall y\ P(y))}\ reintroduction}\ \delta_{\neg\forall}}\ \beta_{\Leftrightarrow}$$

$$\dfrac{P(b) \Leftrightarrow (\forall y\ P(y))}{\dfrac{\boldsymbol{P(b)}, \forall y\ P(y)}{\begin{array}{c}\boldsymbol{\sigma = \{X_2 \mapsto b\}} \\ \boldsymbol{\sigma' = \{X_2 \mapsto sko\}}\end{array}}\ \odot_{\sigma} \qquad \neg P(b), \neg(\forall y\ P(y))}\ \beta_{\Leftrightarrow}$$

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$$P(a), \neg P(b), \forall x \ (P(x) \Leftrightarrow (\forall y \ P(y)))$$

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$$\dfrac{P(b) \Leftrightarrow (\forall y \ P(y))}{P(b), \forall y \ P(y) \quad \Big| \quad \neg P(b), \neg(\forall y \ P(y))} \ \beta_{\Leftrightarrow}$$

$$\dfrac{P(b), \forall y \ P(y)}{\sigma = \{X \mapsto b\}} \ \odot_{\sigma}$$

$$\dfrac{\neg P(b), \neg(\forall y \ P(y))}{\neg P(sko)} \ \delta_{\neg\forall}$$

$$\dfrac{\neg P(sko)}{P(b) \Leftrightarrow (\forall y \ P(y))} \ reintroduction$$

$$\dfrac{P(b) \Leftrightarrow (\forall y \ P(y))}{P(b), \forall y \ P(y) \quad \Big| \quad \neg P(b), \boldsymbol{\neg(\forall y \ P(y))}} \ \beta_{\Leftrightarrow}$$

$$\dfrac{P(b), \forall y \ P(y)}{\sigma = \{X_2 \mapsto b\}} \ \odot_{\sigma} \qquad \sigma' = \{X_2 \mapsto sko\}$$

$$\dfrac{\neg P(b), \boldsymbol{\neg(\forall y \ P(y))}}{\neg P(sko_2)} \ \delta_{\neg\forall}$$

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$$\sigma' = \{X_2 \mapsto sko\}$$

Exploring branches in parallel?

Approach

- Each branch searches for a local solution
- Manages multiple solutions
- No more branch selection fairness problem

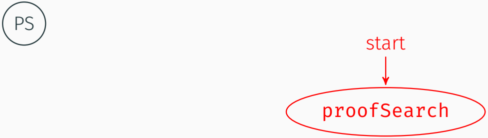
New challenges

- Free variable dependency
- Communication between branches

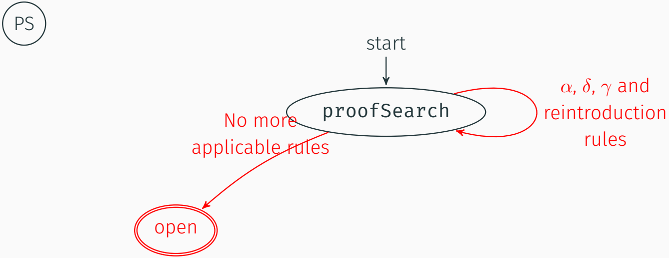
Technical point

- Backtracking on solutions
- Reintroduction fairness problem: iterative deepening

Procedures interactions



Procedures interactions



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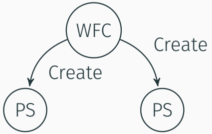

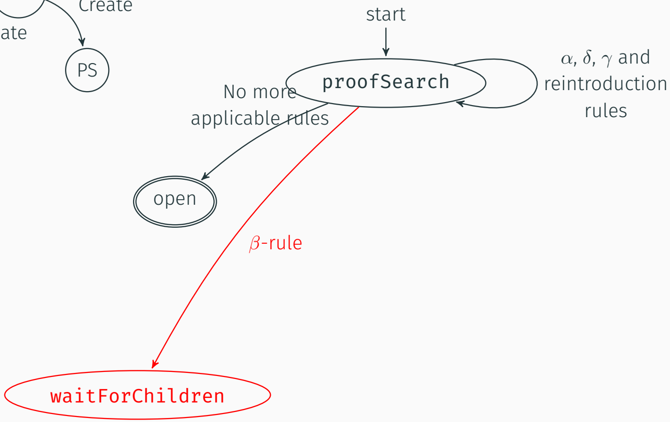

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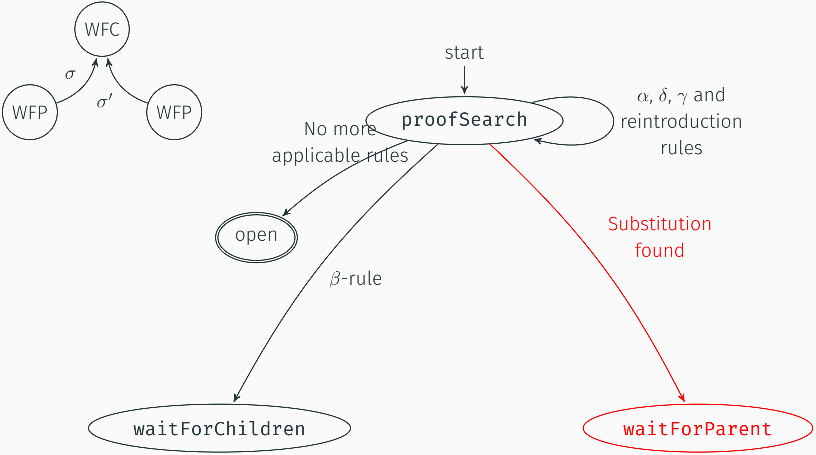

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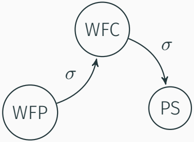

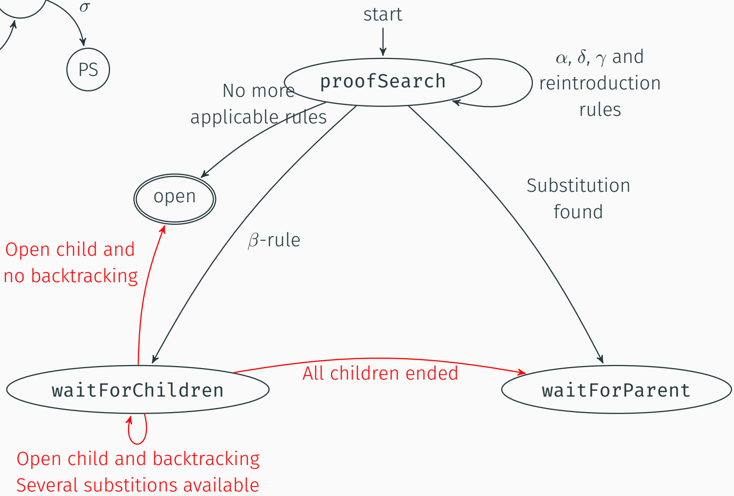

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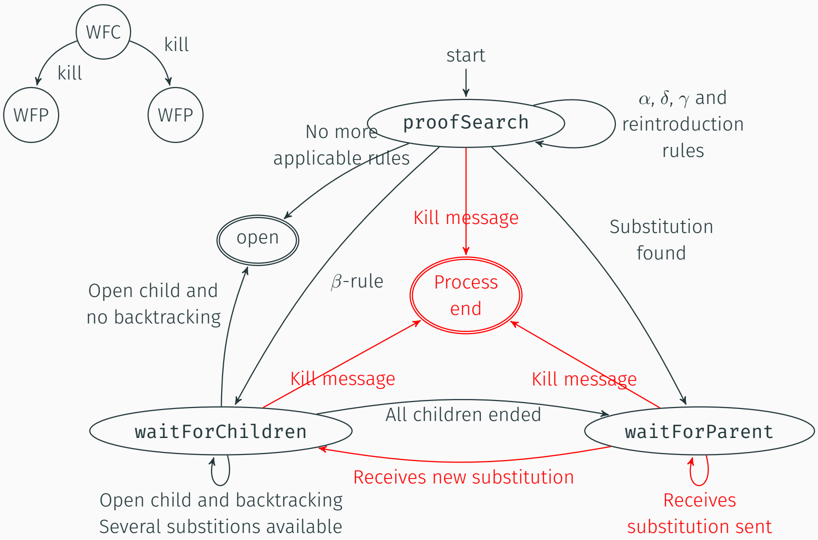

Come back to example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

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$\sigma = \{X \mapsto b\}$ $\qquad$ $\sigma = \{X \mapsto a\}$

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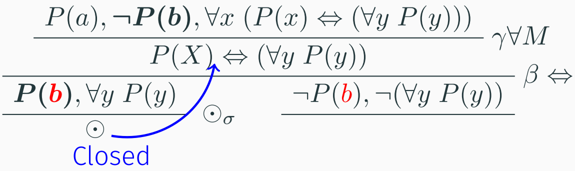

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Open

Come back to example

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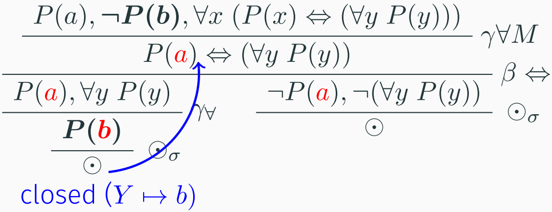


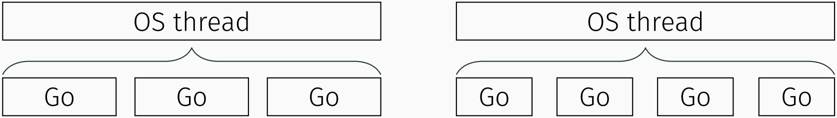
Goéland tool

Functionnalities

- Concurrent proof search algorithm
- Deduction modulo theory (DMT)

Implementation

- Go programming language
- Designed for concurrency
- Goroutines: N:M lightweight threads



Experimentals results on TPTP

	SYN (263 problems)		SET (464 problems)	
Goéland	199		229	
GoélandDMT	199	$(+0, -0)$	272	$(+66, -23)$
Zenon	256	$(+60, -3)$	150	$(+74, -153)$
Princess	195	$(+1, -5)$	258	$(+132, -103)$
LeoIII	195	$(+1, -5)$	177	$(+93, -145)$
E	261	$(+62, -0)$	363	$(+184, -50)$
Vampire	262	$(+63, -0)$	321	$(+167, -75)$

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Analysis and future work

Analysis

- Fairness between branches managed by concurrency
- Promising results for a very new prover, especially with DMT

Future work

- Completeness proof
- Polymorphic types
- Arithmetic (with simplex and branch and bound)

Thank you!

Support Goéland at CASC (15:00!)

`https://github.com/GoelandProver/Goeland`

Tableaux

$$\frac{\bot, \neg\top, P \neg Q}{\odot}\ \odot \qquad \frac{\alpha}{\begin{array}{c}\alpha_1 \\ \alpha_2\end{array}}\ \alpha \qquad \frac{\beta}{\beta_1 \mid \beta_2}\ \beta$$

Where $\sigma(P) = \sigma(Q)$

$$\frac{(\exists/\neg\forall)x.\ \delta(x)}{\delta_1(x \leftarrow f(args))}\ \delta \qquad \frac{(\forall/\neg\exists)x.\ \gamma(x)}{\gamma_1(x \leftarrow X)}\ \gamma$$

Where f is a fresh skolem symbol and $args$ the free variables in δ

Where X is a new variable not occuring anywhere else and waiting for an instanciation

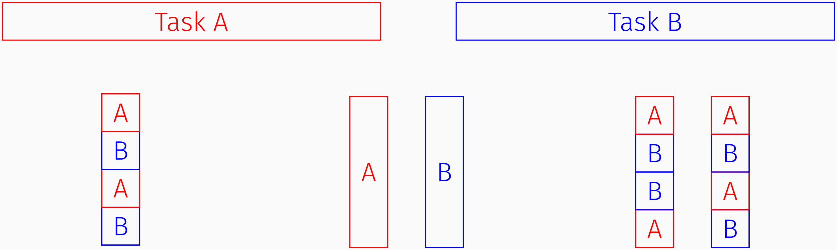
Concurrency vs. parallelism

Concurrency

Concurrency is about an application making progress on more than one task at the same time.

Parallelism

Parallelism is about tasks which can be processed in parallel, for instance on multiple CPUs at the exact same time.



Concurrent but not parallel

Parallel but not concurrent

parallel and concurrent