Deskolemization: From Tableaux to Machine-Checkable Proofs

WG2: Workshop on Automated Reasoning and Proof Logging

EuroProofNet Symposium

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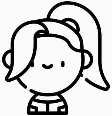
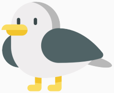
Goéland & Me





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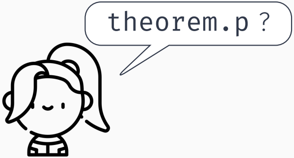

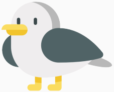

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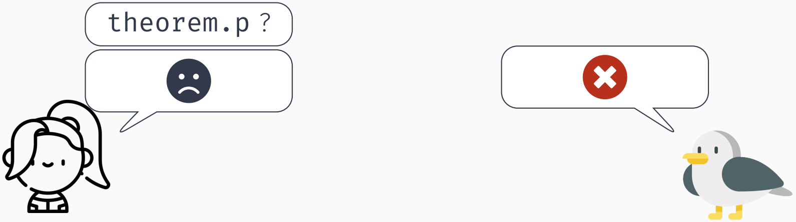
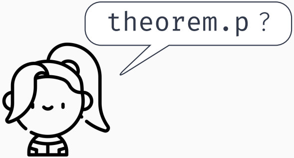
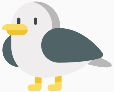



theorem.p ?



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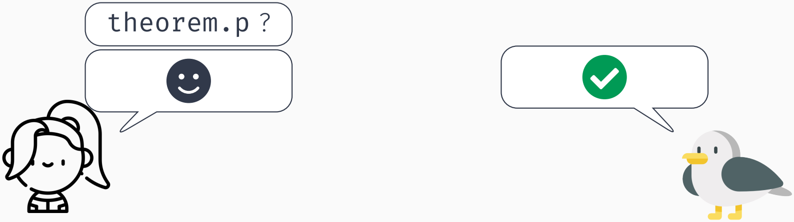

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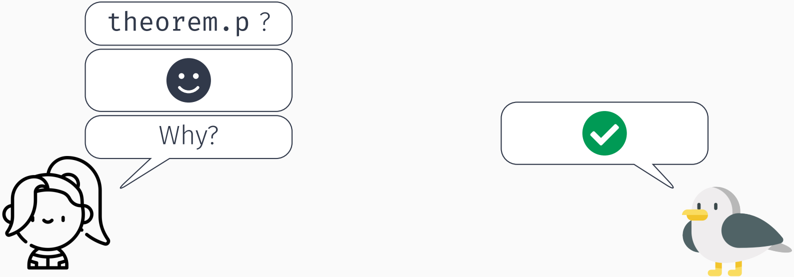

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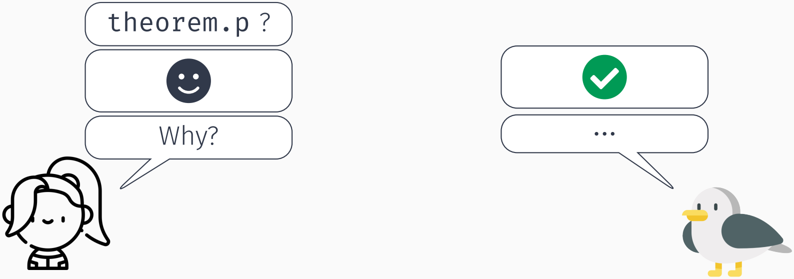

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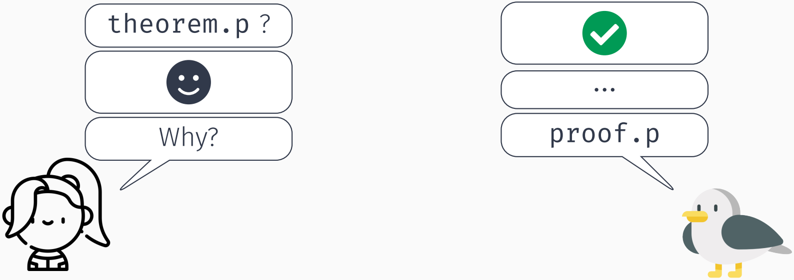

proof.p

```
fof(f4,  assumption, [(a => b), b] --> [((a => b) => (~a | b)), (~a | b), ~a, b],
    inference(hyp, [status(thm), 1, 3], [])).
fof(f3,  assumption, [(a => b), ~a] --> [((a => b) => (~a | b)), (~a | b), ~a, b],
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fof(f2,  plain, [(a => b)] --> [((a => b) => (~a | b)), (~a | b), ~a, b],
    inference(leftImp, [status(thm), 0], [f3, f4])).
fof(f1,  plain, [(a => b)] --> [((a => b) => (~a | b)), (~a | b)],
    inference(rightOr, [status(thm), 1], [f2])).
fof(f0,  plain, [] --> [((a => b) => (~a | b))],
    inference(rightImp, [status(thm), 0], [f1])).
fof(my_conjecture, conjecture, ((a => b) => (~a | b))).
```

proof.p

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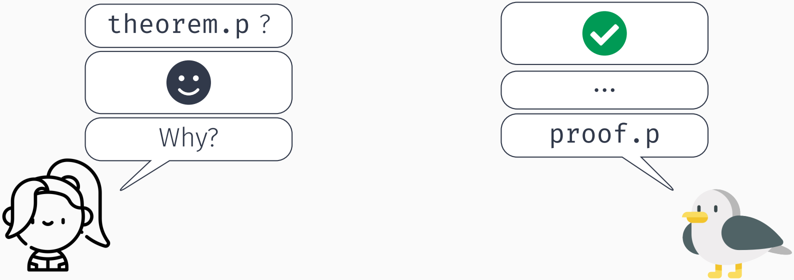
Trust Issues



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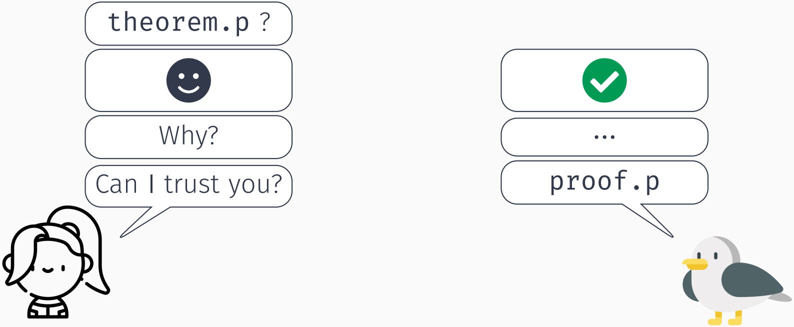
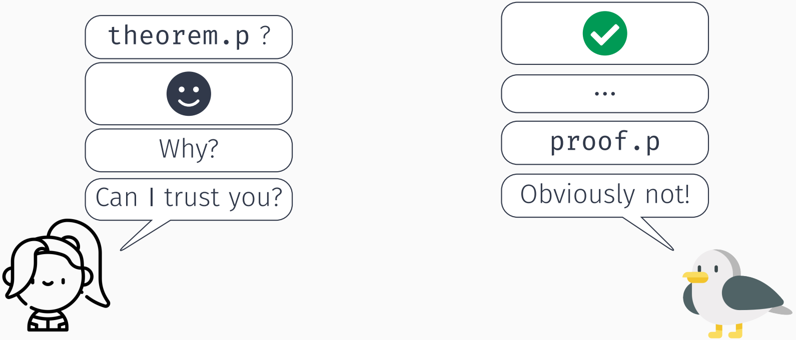


Trust Issues



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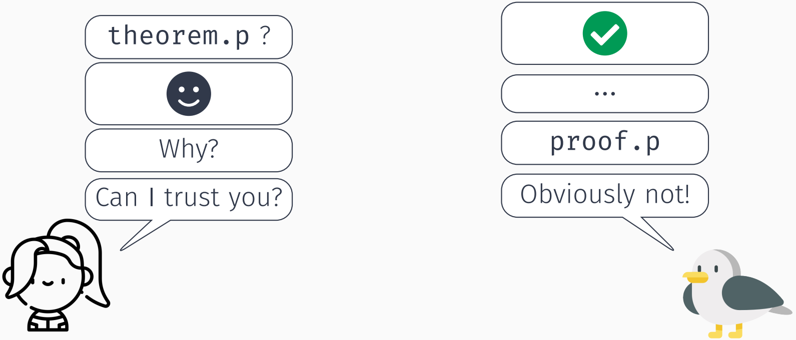

"The only purpose of tableaux is their ability to produce proofs"

— Gilles DOWEK

Tableaux and Sequents

Tableaux

Sequents (GS3)

Tableaux and Sequents

Tableaux

- Original formula

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Tableaux and Sequents

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- Set of inference rules

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1–1 mapping

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Tableaux and Sequents

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$$
\begin{array}{cl}
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) & \\
\hline
\neg(D(X) \Rightarrow \forall y\ D(y)) & \gamma_{\neg\exists} \\
\hline
D(X), \neg(\forall y\ D(y)) & \alpha_{\neg\Rightarrow} \\
\hline
\neg D(f(X)) & \delta_{\neg\forall} \\
\hline
\neg(D(X_2) \Rightarrow \forall y\ D(y)) & \gamma_{\neg\exists} \\
\hline
D(X_2), \neg(\forall y\ D(y)) & \alpha_{\neg\Rightarrow} \\
\hline
\sigma = \{X_2 \mapsto f(X)\} & \odot_\sigma
\end{array}
$$

$$
\begin{array}{cl}
\hline
\ldots, \neg D(c'), D(c'), \neg(\forall y\ D(y)) \vdash & \text{ax} \\
\hline
\ldots, \neg(D(c') \Rightarrow \forall y\ D(y)) \vdash & \neg\Rightarrow \\
\hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \ldots, \neg D(c') \vdash & \neg\exists \\
\hline
\ldots, D(c), \neg(\forall y\ D(y)) \vdash & \neg\forall \\
\hline
\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash & \neg\Rightarrow \\
\hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash & \neg\exists
\end{array}
$$

From GS3 to Rocq

```
Require Export Classical.

Lemma goeland_notnot : forall P : Prop,
  P → (∼ P → False).
Proof. tauto. Qed.

Lemma goeland_nottrue :
  (∼True → False).
Proof. tauto. Qed.

Lemma goeland_and : forall P Q : Prop,
  (P → (Q → False)) → (P ∧ Q → False).
Proof. tauto. Qed.

Lemma goeland_or : forall P Q : Prop,
  (P → False) → (Q → False) → (P ∨ Q → False).
Proof. tauto. Qed.

Lemma goeland_imply : forall P Q : Prop,
  (∼P → False) → (Q → False) → ((P → Q) → False).
Proof. tauto. Qed.

Lemma goeland_equiv : forall P Q : Prop,
  (∼P → ∼Q → False) → (P → Q → False) → ((P ↔
Q) → False).
Proof. tauto. Qed.

Lemma goeland_notand : forall P Q : Prop,
  (∼P → False) → (∼Q → False) → (∼(P ∧ Q) → False).
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```

From GS3 to Rocq

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...
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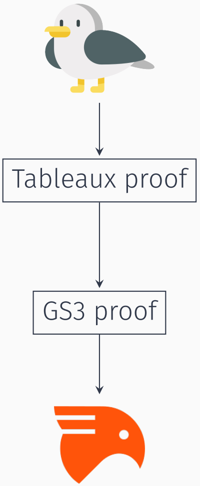
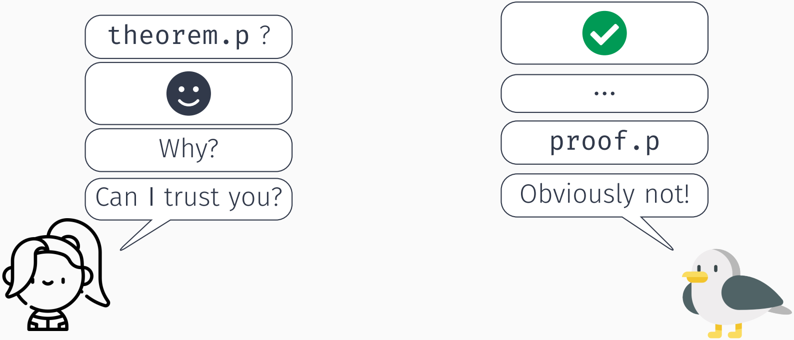


I Trust You!



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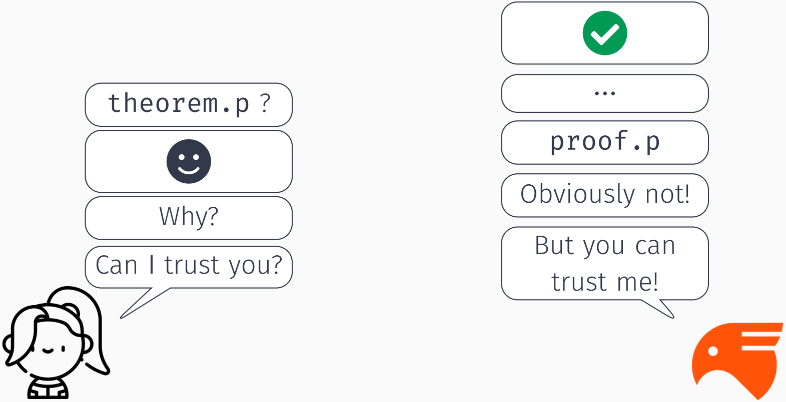

theorem.p ?
Why?
Can I trust you?
...
proof.p
Obviously not!
But you can trust me!

I Trust You!

`theorem.p` ?

Why?

Can I trust you?

Thank you `Rocq`!

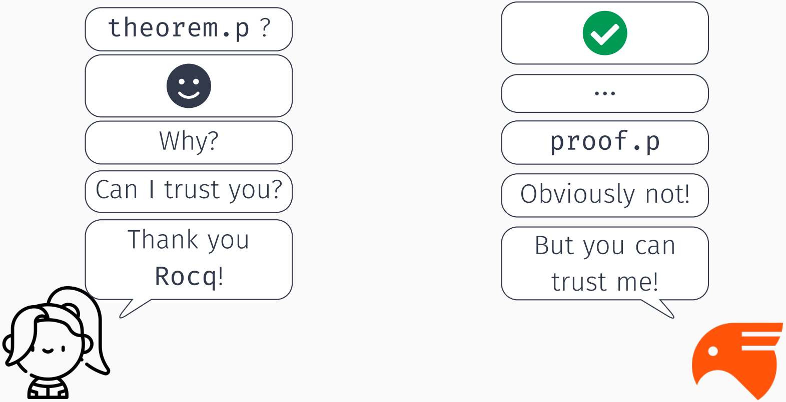
...

`proof.p`

Obviously not!

But you can trust me!

I Trust You!



Unfortunately, it is not that easy...

Once Upon a Proof...

Drinker's Principle

$$\exists x.\ (D(x) \Rightarrow \forall y\ D(y))$$

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$$\cfrac{\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_\sigma \quad \longrightarrow \quad \cfrac{\cfrac{\cfrac{\cfrac{}{\ldots, D(c), \neg(\forall y\ D(y)), \neg D(c) \vdash}\ \mathsf{ax}}{\ldots, D(c), \neg(\forall y\ D(y)) \vdash}\ \neg\forall}{\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg\Rightarrow}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg\exists$$

Once Upon a Proof...

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\quad \longrightarrow \quad
\begin{array}{cl}
 & \\ \hline
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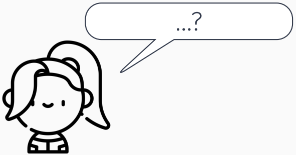
\end{array}$$



Further Investigation Required

Good news:

- This is a correct tableaux proof ☻

Bad news:

- This cannot be turned into a sequent proof ☹

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- Original formula
- Set of inference rules
- Proof search

1–1 mapping

Sequents (GS3)

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Tableaux vs Sequents

Rules

- Closure rules ($\odot$)
- Extension rules (α, β)
- Universal rules (γ)
- Existential rules (δ)

Tableaux vs Sequents

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Tableaux

- Free variables

Sequents

- Final value

$$\dfrac{\dfrac{\neg P(a), \forall x.\ P(x)}{P(X)}\ \gamma_\forall}{\sigma = \{X \mapsto a\}}\ \odot_\sigma$$

(a) Tableaux proof

$$\dfrac{\dfrac{}{\neg P(a), \forall x.\ P(x), P(a) \vdash}\ \mathsf{ax}}{\neg P(a), \forall x.\ P(x) \vdash}\ \forall$$

(b) Sequent proof

Tableaux vs Sequents

Rules

- Closure rules ($\odot$)
- Extension rules (α, β)
- Universal rules (γ)
- Existential rules (δ)

Tableaux

- Fresh Skolem symbol parametrized by free variables

Sequents

- Fresh Skolem symbol

$$\frac{Q(Y,Z), \exists x.\ P(x)}{P(sko(Y,Z))}\ \delta_\exists$$

(a) Tableaux proof

$$\frac{Q(a,b), P(c) \vdash}{Q(a,b), \exists x.\ P(x) \vdash}\ \exists$$

(b) Sequent proof

Which Free Variables?

Flavors of Skolemization

- Outer (δ): the free variables of the branch
- Inner (δ^+): the free variables of the formula
- Pre-inner (δ^{+^+}): δ^+ + reuse Skolem symbols
- δ^*, δ^{*^*}, …

$$\frac{Q(Y, Z), \exists x.\ P(x, Y)}{P(sko(Y, Z), Y)}\ \delta_\exists$$

(a) Outer Skolemization

$$\frac{Q(Y, Z), \exists x.\ P(x, Y)}{P(sko(Y), Y)}\ {\delta^+}_\exists$$

(b) Inner Skolemization

What's Wrong with my Proof?

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma} \quad \rightarrow \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\ldots, D(c), \neg(\forall y\ D(y)), \neg D(c) \vdash}\ \mathsf{ax}}{\ldots, D(c), \neg(\forall y\ D(y)) \vdash}\ \neg_{\forall}\ (\star)}{\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\Rightarrow}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\exists}$$

A Deskolemization Strategy

Idea

Perform all the Skolemization steps before the other rules, so the Skolem symbol is necessarily *fresh*

Key Notions

- Formulas that *depend* on a Skolem symbol
- A formula F needs to be processed before another formula G iff G makes use of a Skolem symbol generated by F

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

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$$\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

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Example

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$$\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)) \vdash}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\Rightarrow}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}\ \neg_{\exists}$$

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Example

$$\cfrac{\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_\sigma$$

$$\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)) \vdash}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)) \vdash}\ \mathsf{W} \times 2}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg\Rightarrow}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}\ \neg\exists$$

Example

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Example

$$
\cfrac{\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}
$$

$$
\cfrac{\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)), \neg D(c) \vdash}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)) \vdash}\ \neg\forall}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)) \vdash}\ \mathsf{W} \times 2}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg\Rightarrow}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}\ \neg\exists
$$

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Example

$$
\begin{array}{cl}
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) & \\ \hline
\neg(D(X) \Rightarrow \forall y\ D(y)) & \gamma_{\neg\exists} \\ \hline
D(X), \neg(\forall y\ D(y)) & \alpha_{\neg\Rightarrow} \\ \hline
\neg D(c) & \delta^{+}_{\neg\forall} \\ \hline
\sigma = \{X \mapsto c\} & \odot_{\sigma}
\end{array}
$$

$$
\begin{array}{cl}
 & \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)), \neg D(c) \vdash & \neg\Rightarrow \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)), \neg D(c) \vdash & \neg\exists \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)) \vdash & \neg\forall \\ \hline\hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)) \vdash & \mathsf{W} \times 2 \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash & \neg\Rightarrow \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) & \neg\exists
\end{array}
$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

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Example

$$
\begin{array}{cl}
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) & \\ \hline
\neg(D(X) \Rightarrow \forall y\ D(y)) & \gamma_{\neg\exists} \\ \hline
D(X), \neg(\forall y\ D(y)) & \alpha_{\neg\Rightarrow} \\ \hline
\neg D(c) & \delta^{+}_{\neg\forall} \\ \hline
\sigma = \{X \mapsto c\} & \odot_{\sigma}
\end{array}
$$

$$
\begin{array}{cl}
\hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)), \neg D(c) \vdash & \text{ax} \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)), \neg D(c) \vdash & \neg\Rightarrow \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)), \neg D(c) \vdash & \neg\exists \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)) \vdash & \neg\forall \\ \hline\hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)) \vdash & \text{W} \times 2 \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash & \neg\Rightarrow \\ \hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) & \neg\exists
\end{array}
$$

A Hydra Game

Beware of the Hydra

- Replaying rules leads to duplicating branches.
- That duplicate the original branch.
- The hydra heads are growing *without* control.
- Without? No: **inter-branches dependency**.

A Hydra Game

Beware of the Hydra

- Replaying rules leads to duplicating branches.
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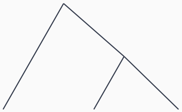
- The hydra heads are growing *without* control.
- Without? No: **inter-branches dependency**.

Kill the Hydra? But it has a Family!

- Should keep formulas when replaying a branching rule.
- But which ones? We provide **conditions**.
- When satisfied, ensure termination and a well-formed proof.
- (weak) requirements on existential rules to work.

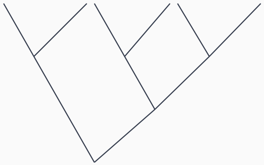
Evaluation Protocol

- SYN and SET categories (TPTP)
- 3 Skolemization strategies + DMT
- Number of problems solved
- `Rocq` output
- Size of the proof (number of branches)
- Average and max size increase



(a) Tableaux proof

→



(b) GS3 translation

Experiments

	Problems Proved	Percentage Certified	Avg. Size Increase	Max. Size Increase
Goéland	261	100 %	0 %	-
Goéland+δ^+	272	100 %	8.1 %	5.3
Goéland+δ^{+^+}	274	100 %	10.6 %	10.3
Goéland+DMT	363	100 %	0 %	-
Goéland+DMT+δ^+	375	100 %	4.5 %	3.9
Goéland+DMT+δ^{+^+}	377	100 %	7.4 %	5.2

Contributions

- A generic deskolemization framework
- Soundness proof
- Instantiation for δ^+ and δ^{+^+} rules in `Goéland`
- Output of GS3 proof into `Rocq`[1], `LambdaPi`[2], `Lisa` and `SC-TPTP`
- Promising results
- 100% of the proofs are certified
- Far below the theoretical bound

[1] If you have questions about this output, ask someone else in this room.
[2] If you have questions about this output, ask someone else not in this room.

Take Home Message

You can perform an efficient (tableaux) proof-search while keeping the ability to produce a (machine-checkable) proof!

What's Next?

- Reduce the number of branches by the use of lemmas
- Integration of theories
- Standalone tool and proof elaboration
- Framework for verification of tableaux proofs: `TableauxRocq`

Thank you! ☺

https://github.com/GoelandProver/Goeland

https://github.com/SC-TPTP/sc-tptp