Goéland: A Concurrent Tableau-Based Theorem Prover

AVM'23

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Context

Method of analytic tableaux

- Free variables
- Usually managed sequentially

Fair proof search is difficult!

- Shared free variables
- Find a subtitution for the whole tree
- Completeness issues: branch selection, free variables reintroduction

Motivating example

$$P(a), \neg P(b), \forall x \; (P(x) \Leftrightarrow (\forall y \; P(y)))$$

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Motivating example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$\frac{P(a), \neg P(b), \boldsymbol{\forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}}{P(X) \Leftrightarrow (\forall y\ P(y))}\ \gamma\forall$$

Motivating example

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$$\dfrac{\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{\boldsymbol{P(X) \Leftrightarrow (\forall y\ P(y))}}\ \gamma_\forall}{P(X), \forall y\ P(y) \qquad \neg P(X), \neg(\forall y\ P(y))}\ \beta_\Leftrightarrow$$

Motivating example

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Motivating example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$\cfrac{\cfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(a) \Leftrightarrow (\forall y\ P(y))}\gamma_\forall}{\cfrac{P(a), \forall \boldsymbol{y}\ \boldsymbol{P(y)}}{P(Y)}\gamma_\forall \qquad \cfrac{\neg P(a), \neg(\forall y\ P(y))}{\sigma = \{X \mapsto a\}}\odot_\sigma}\beta_\Leftrightarrow$$

Motivating example

$$P(a), \neg P(b), \forall x\, (P(x) \Leftrightarrow (\forall y\, P(y)))$$

$$\dfrac{\dfrac{P(a), \neg \boldsymbol{P(b)}, \forall x\, (P(x) \Leftrightarrow (\forall y\, P(y)))}{P({\color{red}a}) \Leftrightarrow (\forall y\, P(y))}\gamma_\forall}{\dfrac{\dfrac{P({\color{red}a}), \forall y\, P(y)}{\boldsymbol{P({\color{red}b})}}\gamma_\forall}{\boldsymbol{\sigma = \{Y \mapsto b\}}}\odot_\sigma \qquad \dfrac{\neg P({\color{red}a}), \neg(\forall y\, P(y))}{\sigma = \{X \mapsto a\}}\odot_\sigma}\beta_\Leftrightarrow$$

Motivating example (other branch)

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

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$$P(a), \neg P(b), \forall x \; (P(x) \Leftrightarrow (\forall y \; P(y)))$$

$$\dfrac{\dfrac{P(a), \boldsymbol{\neg P(b)}, \forall x \; (P(x) \Leftrightarrow (\forall y \; P(y)))}{P(\textcolor{red}{b}) \Leftrightarrow (\forall y \; P(y))} \gamma_\forall}{\dfrac{\boldsymbol{P(\textcolor{red}{b})}, \forall y \; P(y)}{\boldsymbol{\sigma = \{X \mapsto b\}}} \odot_\sigma \qquad \neg P(\textcolor{red}{b}), \neg(\forall y \; P(y))} \beta_\Leftrightarrow$$

Motivating example (other branch)

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Motivating example (other branch)

$$P(a), \neg P(b), \forall x \ (P(x) \Leftrightarrow (\forall y \ P(y)))$$

$$\dfrac{P(a), \neg P(b), \boldsymbol{\forall x \ (P(x) \Leftrightarrow (\forall y \ P(y)))}}{P(\color{red}{b}\color{black}) \Leftrightarrow (\forall y \ P(y))} \ \gamma_{\forall}$$

$$\dfrac{P(\color{red}{b}\color{black}) \Leftrightarrow (\forall y \ P(y))}{P(\color{red}{b}\color{black}), \forall y \ P(y) \qquad \neg P(\color{red}{b}\color{black}), \neg(\forall y \ P(y))} \ \beta_{\Leftrightarrow}$$

$$\dfrac{P(\color{red}{b}\color{black}), \forall y \ P(y)}{\sigma = \{X \mapsto b\}} \ \odot_{\sigma}$$

$$\dfrac{\dfrac{\neg P(\color{red}{b}\color{black}), \neg(\forall y \ P(y))}{\neg P(sko)} \ \delta_{\neg\forall}}{P(X_2) \Leftrightarrow (\forall y \ P(y))} \ reintroduction$$

Motivating example (other branch)

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

$$\dfrac{\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(b) \Leftrightarrow (\forall y\ P(y))}\ \gamma_{\forall}}{\dfrac{P(b), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\ \odot_{\sigma} \qquad \dfrac{\dfrac{\dfrac{\neg P(b), \neg(\forall y\ P(y))}{\neg P(sko)}\ \delta_{\neg\forall}}{\boldsymbol{P(X_2) \Leftrightarrow (\forall y\ P(y))}}\ reintroduction}{P(X_2), \forall y\ P(y) \qquad \neg P(X_2), \neg(\forall y\ P(y))}\ \beta_{\Leftrightarrow}}\ \beta_{\Leftrightarrow}$$

Motivating example (other branch)

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$$\dfrac{\dfrac{P(a), \boldsymbol{\neg P(b)}, \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(b) \Leftrightarrow (\forall y\ P(y))}\ \gamma_{\forall}}{\dfrac{P(b), \forall y\ P(y)}{\sigma = \{X \mapsto b\}}\ \odot_{\sigma} \qquad \dfrac{\dfrac{\dfrac{\neg P(b), \neg(\forall y\ P(y))}{\boldsymbol{\neg P(sko)}}\ \delta_{\neg\forall}}{P(b) \Leftrightarrow (\forall y\ P(y))}\ reintroduction}{\dfrac{\boldsymbol{P(b)}, \forall y\ P(y)}{\begin{array}{c}\boldsymbol{\sigma = \{X_2 \mapsto b\}} \\ \boldsymbol{\sigma' = \{X_2 \mapsto sko\}}\end{array}}\ \odot_{\sigma} \qquad \neg P(b), \neg(\forall y\ P(y))}\ \beta_{\Leftrightarrow}}\ \beta_{\Leftrightarrow}$$

Motivating example (other branch)

$$P(a), \neg P(b), \forall x \; (P(x) \Leftrightarrow (\forall y \; P(y)))$$

$$\dfrac{P(a), \neg P(b), \forall x \; (P(x) \Leftrightarrow (\forall y \; P(y)))}{P(b) \Leftrightarrow (\forall y \; P(y))} \; \gamma_\forall$$

$$\dfrac{P(b) \Leftrightarrow (\forall y \; P(y))}{P(b), \forall y \; P(y) \qquad \neg P(b), \neg(\forall y \; P(y))} \; \beta_\Leftrightarrow$$

$$\dfrac{P(b), \forall y \; P(y)}{\sigma = \{X \mapsto b\}} \; \odot_\sigma$$

$$\dfrac{\neg P(b), \neg(\forall y \; P(y))}{\neg P(sko)} \; \delta_{\neg\forall}$$

$$\dfrac{\neg P(sko)}{P(b) \Leftrightarrow (\forall y \; P(y))} \; reintroduction$$

$$\dfrac{P(b) \Leftrightarrow (\forall y \; P(y))}{P(b), \forall y \; P(y) \qquad \neg P(b), \neg(\forall \boldsymbol{y} \; \boldsymbol{P(y)})} \; \beta_\Leftrightarrow$$

$$\dfrac{P(b), \forall y \; P(y)}{\sigma = \{X_2 \mapsto b\}} \; \odot_\sigma \qquad \sigma' = \{X_2 \mapsto sko\}$$

$$\dfrac{\neg P(b), \neg(\forall \boldsymbol{y} \; \boldsymbol{P(y)})}{\neg P(sko_2)} \; \delta_{\neg\forall}$$

Motivating example (other branch)

$$P(a), \neg P(b), \forall x \; (P(x) \Leftrightarrow (\forall y \; P(y)))$$

$$\dfrac{P(a), \neg P(b), \forall x \; (P(x) \Leftrightarrow (\forall y \; P(y)))}{P(b) \Leftrightarrow (\forall y \; P(y))} \; \gamma_{\forall}$$

$$\dfrac{P(b) \Leftrightarrow (\forall y \; P(y))}{P(b), \forall y \; P(y) \qquad \neg P(b), \neg(\forall y \; P(y))} \; \beta_{\Leftrightarrow}$$

$$\dfrac{P(b), \forall y \; P(y)}{\sigma = \{X \mapsto b\}} \; \odot_{\sigma}$$

$$\dfrac{\neg P(b), \neg(\forall y \; P(y))}{\neg P(sko)} \; \delta_{\neg\forall}$$

$$\dfrac{\neg P(sko)}{P(b) \Leftrightarrow (\forall y \; P(y))} \; reintroduction$$

$$\dfrac{P(b) \Leftrightarrow (\forall y \; P(y))}{P(b), \forall y \; P(y) \qquad \neg P(b), \neg(\forall y \; P(y))} \; \beta_{\Leftrightarrow}$$

$$\dfrac{P(b), \forall y \; P(y)}{\sigma = \{X_2 \mapsto b\}} \; \odot_{\sigma}$$

$$\sigma' = \{X_2 \mapsto sko\}$$

$$\dfrac{\neg P(b), \neg(\forall y \; P(y))}{\neg P(sko_2)} \; \delta_{\neg\forall}$$

$$\dfrac{\neg P(sko_2)}{\ldots} \; reintroduction$$

Exploring branches in parallel?

Approach

- Each branch searches for a local solution
- Manages multiple solutions
- No more branch selection fairness problem

New challenges

- Free variable dependency
- Communication between branches

Technical point

- Backtracking on solutions
- Reintroduction fairness problem: iterative deepening

Come back to example

$$P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))$$

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Come back to example

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$$\sigma = \{X \mapsto b\} \qquad \sigma = \{X \mapsto a\}$$

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Closed

Come back to example

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$$\dfrac{\dfrac{P(a), \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(X) \Leftrightarrow (\forall y\ P(y))}\ \gamma\forall M}{\dfrac{P({\color{red}b}), \forall y\ P(y)}{\odot}\ \odot_{\sigma} \qquad \dfrac{\neg P({\color{red}b}), \boldsymbol{\neg(\forall y\ P(y))}}{P(sko)}\ \delta_{\neg\forall}}\ \beta \Leftrightarrow$$

Come back to example

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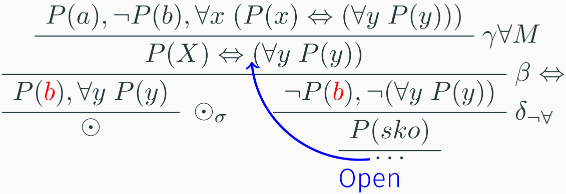

Come back to example

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$$\color{blue}{\sigma = \{X \mapsto a\}} \quad\quad \color{blue}{\sigma = \{X \mapsto a\}}$$

Come back to example

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$$\dfrac{\mathbf{P(a)}, \neg P(b), \forall x\ (P(x) \Leftrightarrow (\forall y\ P(y)))}{P(X) \Leftrightarrow (\forall y\ P(y))}\ \gamma\forall M$$

$$\dfrac{P(X) \Leftrightarrow (\forall y\ P(y))}{P(a), \forall y\ P(y) \qquad \dfrac{\neg \mathbf{P(a)}, \neg(\forall y\ P(y))}{\odot}\ \odot_\sigma}\ \beta \Leftrightarrow$$

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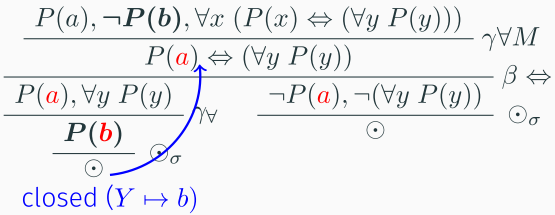


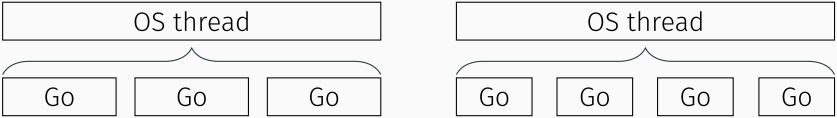
Goéland tool

Functionnalities

- Concurrent proof search algorithm
- Deduction modulo theory (DMT)

Implementation

- Go programming language
- Designed for concurrency
- Goroutines: N:M lightweight threads



Experimentals results on TPTP

	SYN (263 problems)		SET (464 problems)	
Goéland	199		229	
GoélandDMT	199	$(+0, -0)$	272	$(+66, -23)$
Zenon	256	$(+60, -3)$	150	$(+74, -153)$
Princess	195	$(+1, -5)$	258	$(+132, -103)$
LeoIII	195	$(+1, -5)$	177	$(+93, -145)$
E	261	$(+62, -0)$	363	$(+184, -50)$
Vampire	262	$(+63, -0)$	321	$(+167, -75)$

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Analysis and future work

Analysis

- Fairness between branches managed by concurrency
- Promising results for a very new prover, especially with DMT

Future work

- Completeness proof
- Polymorphic types
- Equality Reasoning
- Arithmetic (with simplex and branch and bound)
- ITP output

Thank you!

`https://github.com/GoelandProver/Goeland`