Deskolemization: From Tableaux to Proof Certificates

CHoCoLa Meeting

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December 11, 2025

Computer-Assisted Proofs

✔ Interactive Theorem Proving (ITP)

- Proof assistants
- Guided search
- Proof certificate
- Nice logos

...

Computer-Assisted Proofs

Automated Theorem Proving (ATP)

- Click-and-prove software
- Autonomous search
- Statement or proof-like trace
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Leo-III

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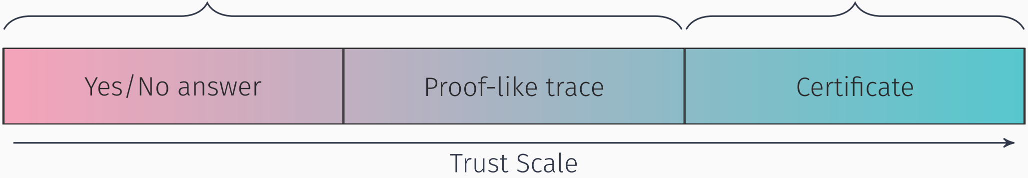
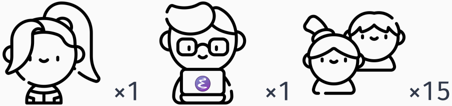
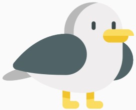


In this Talk...

Automated Reasoning in FOL

- First-order classical logic
- Analytic tableaux



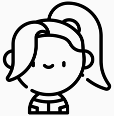


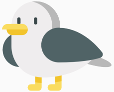
The `Goéland` Tool

- A concurrent theorem prover
- 40 000 lines of code (**Go**)
- 345L of interns' tears
- Equality reasoning, deduction modulo theory, second-class polymorphism[1], ...

[1]Thank you, Johann!

Goéland & Me





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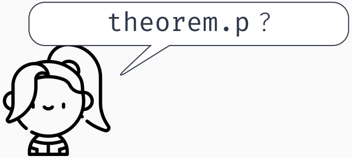

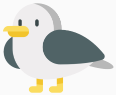

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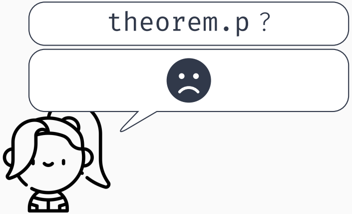
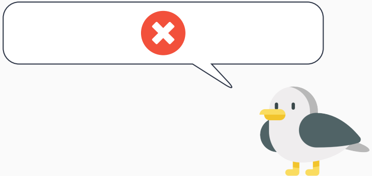
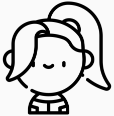
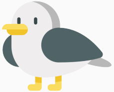




Goéland & Me

Some debugging later...





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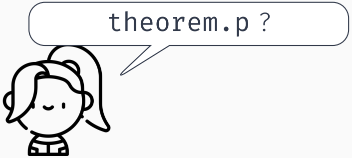

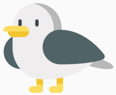


theorem.p ?

Goéland & Me

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proof.p

```
fof(f4,  assumption, [(a => b), b] --> [((a => b) => (~a | b)), (~a | b), ~a, b],
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fof(f2,  plain, [(a => b)] --> [((a => b) => (~a | b)), (~a | b), ~a, b],
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fof(f1,  plain, [(a => b)] --> [((a => b) => (~a | b)), (~a | b)],
    inference(rightOr, [status(thm), 1], [f2])).

fof(f0,  plain, [] --> [((a => b) => (~a | b))],
    inference(rightImp, [status(thm), 0], [f1])).

fof(my_conjecture, conjecture, ((a => b) => (~a | b))).
```

proof.p

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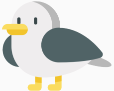
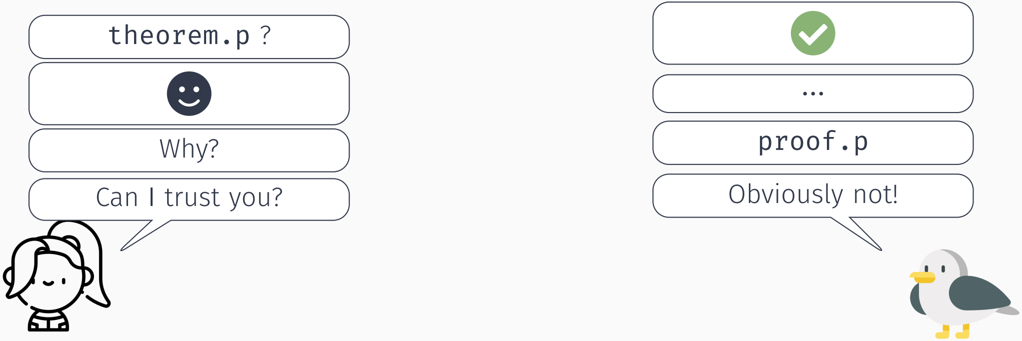
Trust Issues

Trust Issues



Trust Issues



theorem.p ?

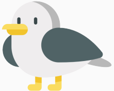
Why?

Can I trust you?

…

proof.p

Obviously not!



"The only purpose of tableaux is their ability to produce proofs"

— Gilles DOWEK

Tableaux and Sequents

Tableaux

Sequents (GS3)

Tableaux and Sequents

Tableaux

- Original formula

Sequents (GS3)

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Tableaux and Sequents

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- Set of inference rules

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1–1 mapping

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Tableaux and Sequents

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$$\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))$$
$$\gamma_{\neg\exists}$$
$$\neg(D(X) \Rightarrow \forall y\ D(y))$$
$$\alpha_{\neg\Rightarrow}$$
$$D(X), \neg(\forall y\ D(y))$$
$$\delta_{\neg\forall}$$
$$\neg D(f(X))$$
$$\gamma_{\neg\exists}$$
$$\neg(D(X_2) \Rightarrow \forall y\ D(y))$$
$$\alpha_{\neg\Rightarrow}$$
$$D(X_2), \neg(\forall y\ D(y))$$
$$\odot_\sigma$$
$$\sigma = \{X_2 \mapsto f(X)\}$$

$$\text{ax}$$
$$\ldots, \neg D(c'), D(c'), \neg(\forall y\ D(y)) \vdash$$
$$\neg_\Rightarrow$$
$$\ldots, \neg(D(c') \Rightarrow \forall y\ D(y)) \vdash$$
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$$\neg_\exists$$
$$\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash$$

From Sequents to Rocq

```
Require Export Classical.

Lemma goeland_notnot : forall P : Prop,
  P → (~ P → False).
Proof. tauto. Qed.

Lemma goeland_nottrue :
  (~True → False).
Proof. tauto. Qed.

Lemma goeland_and : forall P Q : Prop,
  (P → (Q → False)) → (P ∧ Q → False).
Proof. tauto. Qed.

Lemma goeland_or : forall P Q : Prop,
  (P → False) → (Q → False) → (P ∨ Q → False).
Proof. tauto. Qed.

Lemma goeland_imply : forall P Q : Prop,
  (~P → False) → (Q → False) → ((P → Q) → False).
Proof. tauto. Qed.

Lemma goeland_equiv : forall P Q : Prop,
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Proof. tauto. Qed.

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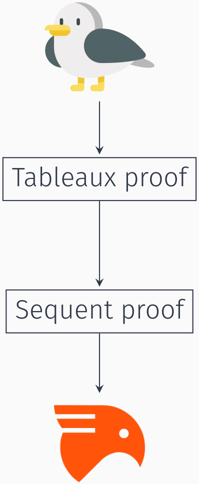
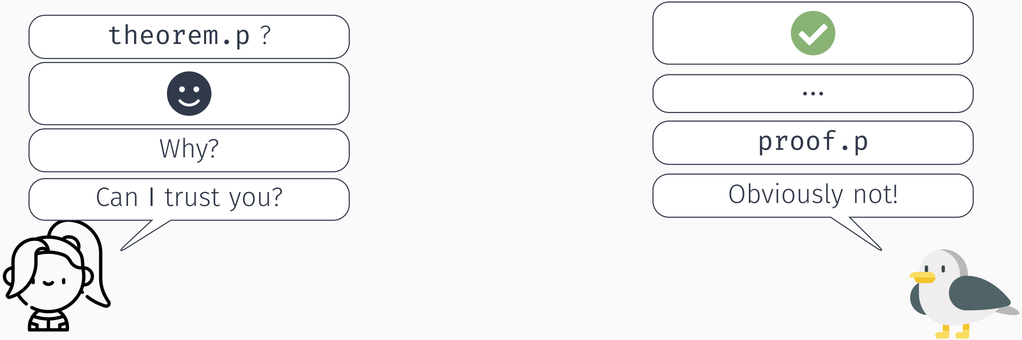


I Trust You!



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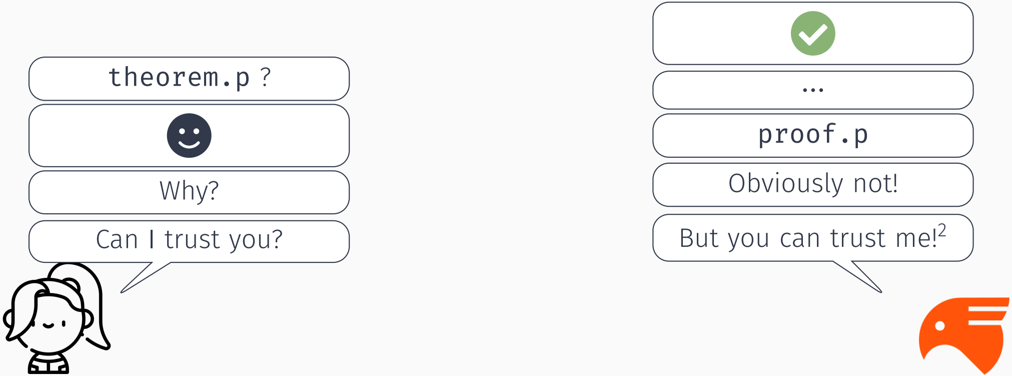

[2]Even if some people in this room can prouve **False** in **Rocq** in 10 lines.

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`theorem.p` ?

Why?

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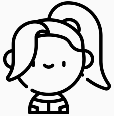
Thank you **Rocq**!



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Obviously not!

But you can trust me![2]

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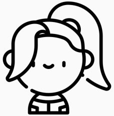

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Obviously not!

But you can trust me![2]

> Unfortunately, life is not that easy...

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Once Upon a Proof...

Drinker's Principle

$$\exists x. (D(x) \Rightarrow \forall y\, D(y))$$

Drinker's Principle

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\quad \rightsquigarrow \quad
\dfrac{\dfrac{\dfrac{\dfrac{}{\ldots, D(c), \neg(\forall y\ D(y)), \neg D(c) \vdash}\ \mathrm{ax}}{\ldots, D(c), \neg(\forall y\ D(y)) \vdash}\ \neg_{\forall}}{\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\Rightarrow}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\exists}$$

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$$\rightsquigarrow$$

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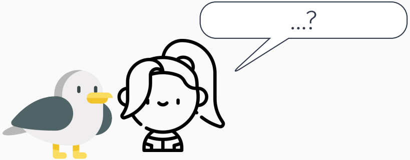

Further Investigation Required

Good news:

- This is a correct tableaux proof ☻

Bad news:

- This cannot be turned into a sequent proof with our current implementation ☹

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Tableaux

- Original formula
- Set of inference rules
- Proof search

1–1 mapping (Set of inference rules ↔ Set of inference rules)

Sequents (GS3)

- Original formula
- Set of inference rules
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Tableaux vs Sequents

Rules

- Closure rules ($\odot$)
- Extension rules (α, β)
- Universal rules (γ)
- Existential rules (δ)

Tableaux vs Sequents

Rules

- Closure rules ($\odot$)
- Extension rules (α, β)
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Tableaux

- Free variables

Sequents

- Final value

$$\cfrac{\cfrac{\neg P(a), \forall x.\ P(x)}{P(X)}\ \gamma_\forall}{\sigma = \{X \mapsto a\}}\ \odot_\sigma$$

(a) Tableaux proof

$$\cfrac{\cfrac{}{\neg P(a), \forall x.\ P(x), P(a) \vdash}\ \text{ax}}{\neg P(a), \forall x.\ P(x) \vdash}\ \forall$$

(b) Sequent proof

Tableaux vs Sequents

Rules

- Closure rules ($\odot$)
- Extension rules (α, β)
- Universal rules (γ)
- Existential rules (δ)

Tableaux

- Fresh Skolem symbol parametrized by the free variables of the branch

Sequents

- Fresh Skolem symbol

$$\frac{Q(Y,Z), \exists x.\ P(x)}{P(sko(Y,Z))}\ \delta_\exists$$

(a) Tableaux proof

$$\frac{Q(a,b), P(c) \vdash}{Q(a,b), \exists x.\ P(x) \vdash}\ \exists$$

(b) Sequent proof

All the Free Variables?

Flavors of Skolemization

- Outer (δ): the free variables of the branch
- Inner (δ^{+}): the free variables of the formula
- Pre-inner ($\delta^{+^{+}}$): δ^{+} + reuse Skolem symbols
- δ^{*}, $\delta^{*^{*}}$, ...
- Shorter proofs and faster proof search!

$$\frac{Q(Y,Z), \exists x.\ P(x,Y)}{P(sko(Y,Z),Y)}\ \delta_{\exists}$$

(a) Outer Skolemization

$$\frac{Q(Y,Z), \exists x.\ P(x,Y)}{P(sko(Y),Y)}\ \delta^{+}_{\exists}$$

(b) Inner Skolemization

What’s Wrong with my Proof?

$$
\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\cfrac{\neg(D(X) \Rightarrow \forall y\ D(y))}{\cfrac{D(X), \neg(\forall y\ D(y))}{\cfrac{\neg D(c)}{\sigma = \{X \mapsto c\}}\ \odot_\sigma}\ \delta^+_{\neg\forall}}\ \alpha_{\neg\Rightarrow}}\ \gamma_{\neg\exists}
$$

$\rightsquigarrow$

$$
\cfrac{\cfrac{\cfrac{\cfrac{}{\ldots, D(c), \neg(\forall y\ D(y)), \neg D(c) \vdash}\ \mathrm{ax}}{\ldots, D(c), \neg(\forall y\ D(y)) \vdash}\ \neg_\forall\ (\star)}{\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_\Rightarrow}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_\exists
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X),\ \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}
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$$

There is Hope!

Outer Skolemization is equivalent to the corresponding sequent rule!

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$$
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(f(X))}\ \delta_{\neg\forall}}{\neg(D(X_2) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X_2), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\sigma = \{X_2 \mapsto f(X)\}}\ \odot_{\sigma}
$$

(a) Outer Skolemization tableau proof

$$
\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{}{\ldots, \neg D(c'), D(c'), \neg(\forall y\ D(y)) \vdash}\ \mathsf{ax}}{\ldots, \neg(D(c') \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\Rightarrow}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \ldots, \neg D(c') \vdash}\ \neg_{\exists}}{\ldots, D(c), \neg(\forall y\ D(y)) \vdash}\ \neg_{\forall}}{\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\Rightarrow}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\exists}
$$

(b) Equivalent sequent

There is Hope!

Outer Skolemization is equivalent to the corresponding sequent rule!

$$
\begin{array}{cl}
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) & \\
\hline
\neg(D(X) \Rightarrow \forall y\ D(y)) & \gamma_{\neg\exists} \\
\hline
D(X), \neg(\forall y\ D(y)) & \alpha_{\neg\Rightarrow} \\
\hline
\neg D(f(X)) & \delta_{\neg\forall} \\
\hline
\neg(D(X_2) \Rightarrow \forall y\ D(y)) & \gamma_{\neg\exists} \\
\hline
D(X_2), \neg(\forall y\ D(y)) & \alpha_{\neg\Rightarrow} \\
\hline
\sigma = \{X_2 \mapsto f(X)\} & \odot_{\sigma}
\end{array}
$$

(a) Outer Skolemization tableau proof

$$
\begin{array}{cl}
\hline
\ldots, \neg D(c'), D(c'), \neg(\forall y\ D(y)) \vdash & \text{ax} \\
\hline
\ldots, \neg(D(c') \Rightarrow \forall y\ D(y)) \vdash & \neg_{\Rightarrow} \\
\hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \ldots, \neg D(c') \vdash & \neg_{\exists} \\
\hline
\ldots, D(c), \neg(\forall y\ D(y)) \vdash & \neg_{\forall} \\
\hline
\ldots, \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash & \neg_{\Rightarrow} \\
\hline
\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash & \neg_{\exists}
\end{array}
$$

(b) Equivalent sequent

The Best of Both Worlds

Can we still make use of advanced skolemization strategies while keeping the ability to turn tableaux proofs intro sequents ones?

The Best of Both Worlds: Deskolemization!

Can we still make use of advanced skolemization strategies while keeping the ability to turn tableaux proofs intro sequents ones?

Yes!

Deskolemization

Perform all Skolemization steps before the other rules, so the Skolem symbol is necessarily *fresh*

Key Notions

- Formulas that *depend* on a Skolem symbol
- A formula F needs to be processed before another formula G iff G makes use of a Skolem symbol generated by F

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

$$\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

$$\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y))}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\exists}$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

$$\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y))}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\exists}$$

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

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Example

$$\cfrac{\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

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Example

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Example

$$
\dfrac{\dfrac{\dfrac{\dfrac{\neg(\exists x.\, D(x) \Rightarrow \forall y\, D(y))}{\neg(D(X) \Rightarrow \forall y\, D(y))}\gamma_{\neg\exists}}{D(X), \neg(\forall y\, D(y))}\alpha_{\neg\Rightarrow}}{\neg D(c)}\delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\odot_{\sigma}
$$

$$
\dfrac{\dfrac{\dfrac{\neg(\exists x.\, D(x) \Rightarrow \forall y\, D(y)), \neg(\forall y\, D(y)) \vdash}{\neg(\exists x.\, D(x) \Rightarrow \forall y\, D(y)), \neg(D(c) \Rightarrow \forall y\, D(y)), D(c), \neg(\forall y\, D(y)) \vdash}\mathrm{W} \times 2}{\neg(\exists x.\, D(x) \Rightarrow \forall y\, D(y)), \neg(D(c) \Rightarrow \forall y\, D(y)) \vdash}\neg_{\Rightarrow}}{\neg(\exists x.\, D(x) \Rightarrow \forall y\, D(y)) \vdash}\neg_{\exists}
$$

Example

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Example

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$$\dfrac{\dfrac{\dfrac{\dfrac{}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(\forall y\ D(y)) \vdash}\ \neg_{\forall}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)) \vdash}\ W \times 2}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\Rightarrow}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\exists}$$

Example

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Example

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Example

$$\cfrac{\cfrac{\cfrac{\cfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y))}{\neg(D(X) \Rightarrow \forall y\ D(y))}\ \gamma_{\neg\exists}}{D(X), \neg(\forall y\ D(y))}\ \alpha_{\neg\Rightarrow}}{\neg D(c)}\ \delta^{+}_{\neg\forall}}{\sigma = \{X \mapsto c\}}\ \odot_{\sigma}$$

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$$\dfrac{\neg(\exists x.\, D(x) \Rightarrow \forall y\, D(y))}{\neg(D(X) \Rightarrow \forall y\, D(y))}\;\gamma_{\neg\exists}$$

$$\dfrac{\neg(D(X) \Rightarrow \forall y\, D(y))}{D(X), \neg(\forall y\, D(y))}\;\alpha_{\neg\Rightarrow}$$

$$\dfrac{D(X), \neg(\forall y\, D(y))}{\neg D(c)}\;\delta^{+}_{\neg\forall}$$

$$\dfrac{\neg D(c)}{\sigma = \{X \mapsto c\}}\;\odot_{\sigma}$$

$$\dfrac{\neg(\exists x.\, D(x) \Rightarrow \forall y\, D(y)), \neg(D(c) \Rightarrow \forall y\, D(y)), \neg(\forall y\, D(y)), \neg D(c) \vdash}{\neg(\exists x.\, D(x) \Rightarrow \forall y\, D(y)), \neg(\forall y\, D(y)), \neg D(c) \vdash}\;\neg_{\exists}$$

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Example

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$$\dfrac{\dfrac{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)), D(c), \neg(\forall y\ D(y)) \vdash}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)), \neg(D(c) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\Rightarrow}}{\neg(\exists x.\ D(x) \Rightarrow \forall y\ D(y)) \vdash}\ \neg_{\exists}$$

Example

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A Hydra Game

Beware of the Hydra

- Replaying rules leads to duplicating branches
- That duplicate the original branch
- The hydra heads are growing *without* control
- Without? No: **inter-branches dependency**

A Hydra Game

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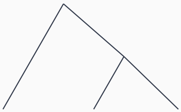
Kill the Hydra? But it has a Family!

- Should keep formulas when replaying a branching rule
- But which ones? We provide **conditions**
- When satisfied, ensure termination and a well-formed proof
- (weak) requirements on existential rules to work

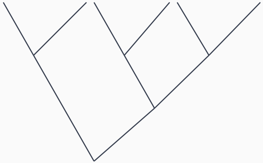
Evaluation Protocol

- SYN and SET categories (TPTP)
- 3 Skolemization strategies + DMT
- Number of problems solved
- Rocq output
- Size of the proof (number of branches)
- Average and max size increase



(a) Tableaux proof

⇝



(b) GS3 translation

Experiments

	Problems Proved	Percentage Certified	Avg. Size Increase	Max. Size Increase
Goéland	261	100 %	0 %	-
Goéland+δ^+	272	100 %	8.1 %	5.3
Goéland+δ^{+^+}	274	100 %	10.6 %	10.3
Goéland+DMT	363	100 %	0 %	-
Goéland+DMT+δ^+	375	100 %	4.5 %	3.9
Goéland+DMT+δ^{+^+}	377	100 %	7.4 %	5.2

Contributions

- A generic deskolemization framework
- Soundness proof
- Instantiation for δ^{+} and $\delta^{+^{*}}$ rules in `Goéland`
- Output of GS3 proof into `Rocq`, `LambdaPi` and `Lisa`
- Promising results
- 100% of the proofs are certified
- Far below the theoretical bound

Contributions

- A generic deskolemization framework
- Soundness proof
- Instantiation for δ^+ and δ^{+^+} rules in `Goéland`
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What About Us?

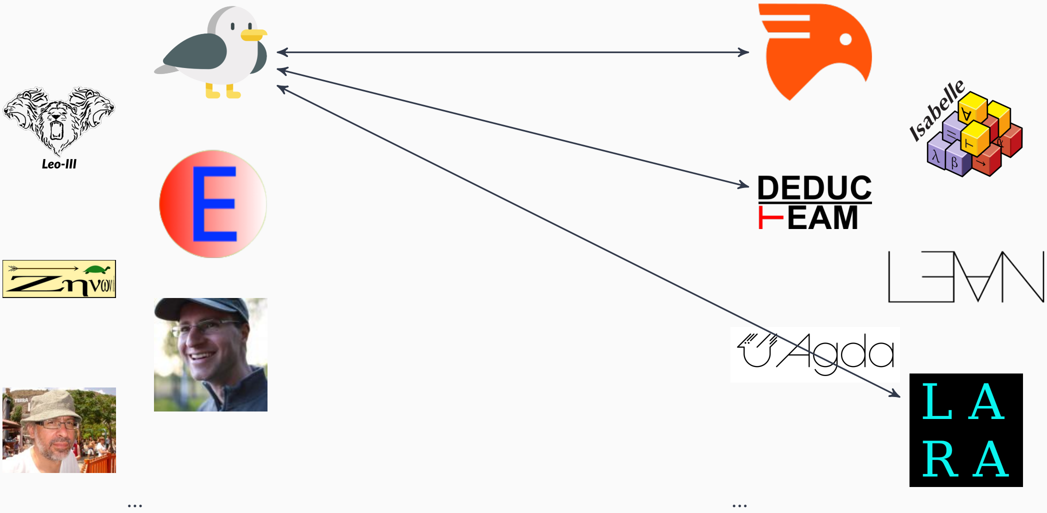
Leo-III LEAN Zenon   Isabelle E Agda ...

Proof Transfers

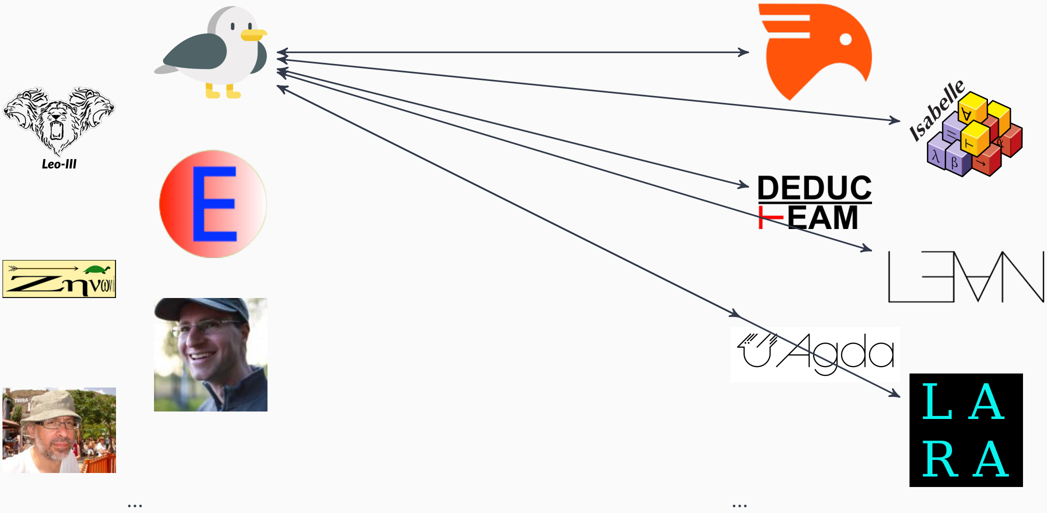
...

...

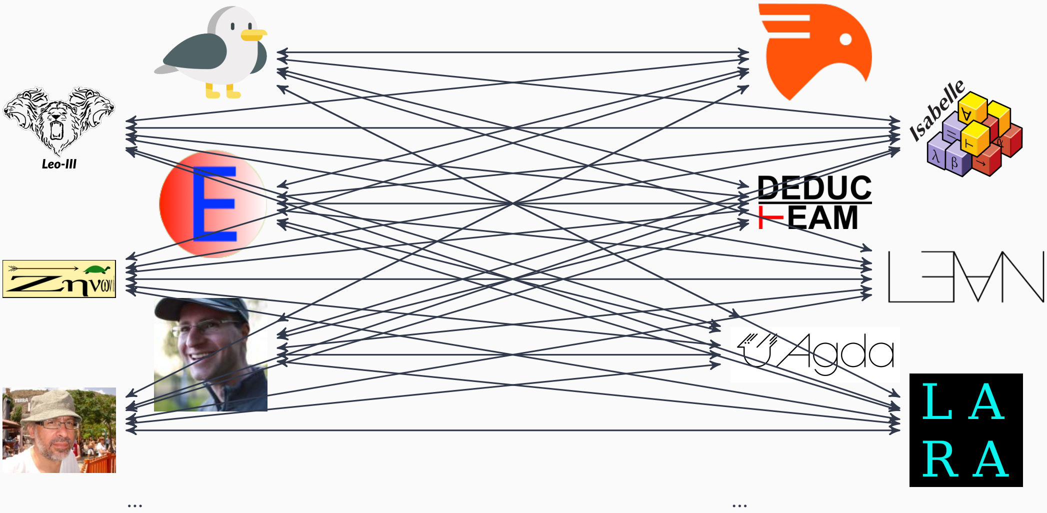
Proof Transfers

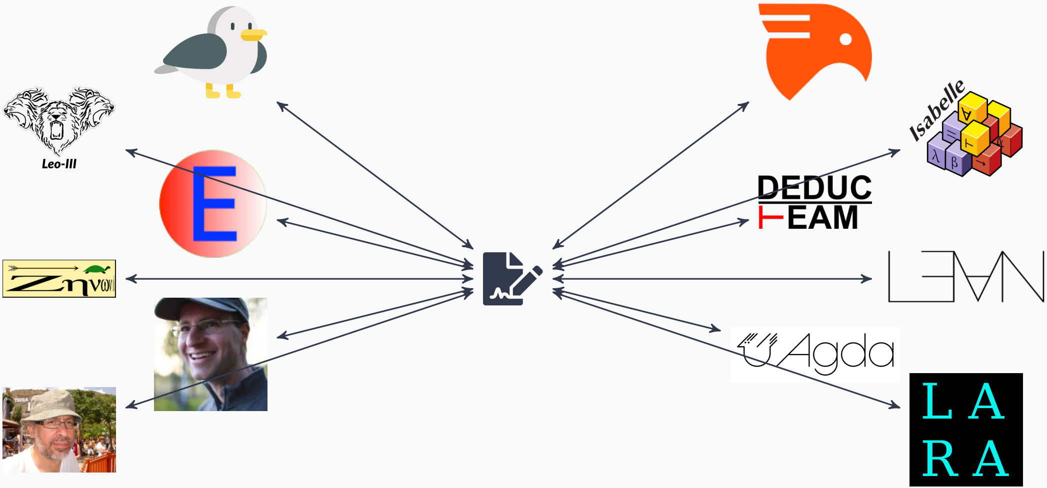


Proof Transfers

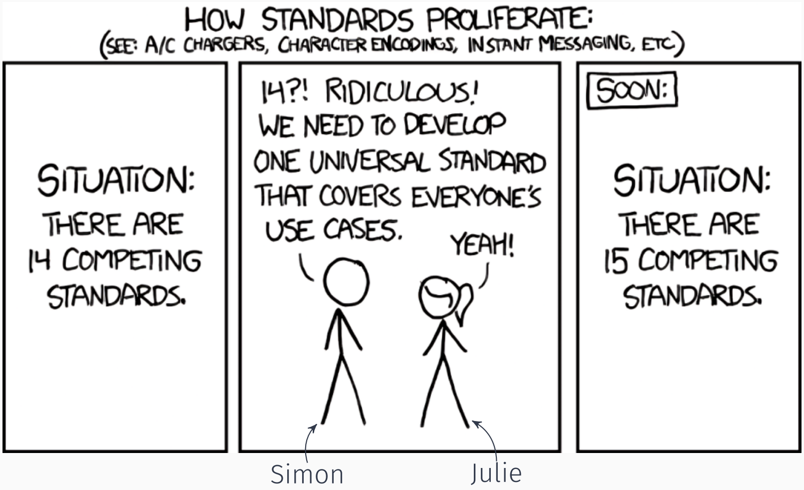


Proof Transfers




Leo-III
Isabelle
DEDUC
⊢EAM
Agda
L A
R A

... ...



Credit: xkcd (`https://xkcd.com/`)

The SC-TPTP Format

The Format

- Extension of **TPTP** for sequent-based calculus
- Standard input format for ATP
- Easy syntax

Rules

- List of supported rules
- Level of steps
- Management of non-deductive steps

```
fof(<name>, <role>, [<formula_list>] --> [<formula_list>],
<annotations>).
```

```
fof(f2, plain, [a | b, b] --> [], ...).
fof(f1, plain, [a | b, a] --> [], ...).
fof(f0, plain, [a | b] --> [], inference(leftOr, [status(thm), 0], [f1,
f2])).
```

SC-TPTP vs. LambdaPi

LambdaPi

- ITP-oriented
- Handle any foundation
- Hard to parse/import
- Translation from many proof assistants (+ some ATP/SMT solvers)

SC-TPTP

- ATP-oriented
- Focus on proof exchanges
- Easy to parse & reconstruct
- Close to current ATP's output format

SC-TPTP Tools

SC-TPTP Utils

- Proof checker
- Proof transformation

Compatible Tools

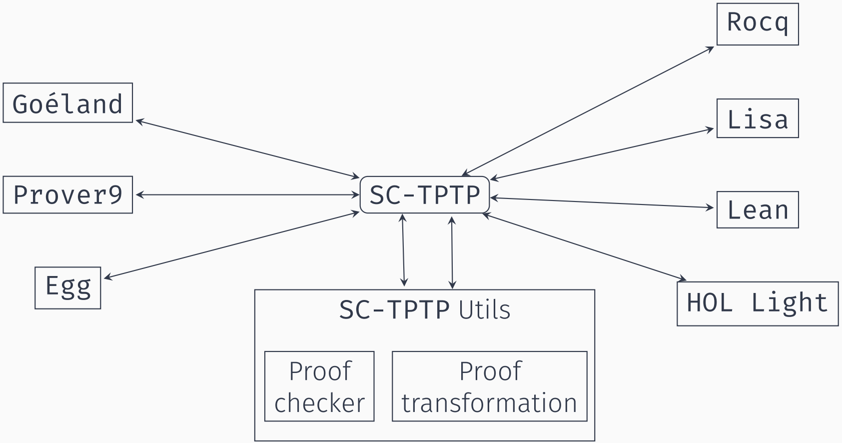
- Proof-producing ATP
- Tactics for ITP

```
example ( α : Type) [Nonempty α] (d : α → Prop) :
∃ y : α, ∀ x : α, (d x → d y) := by goeland
```

```
val drinkers2 = Theorem(∃(x, ∀(y, d(x) ==> d(y)))):
have(thesis) by Goeland

val thm = Theorem((∀(x, P(x)) \/ ∀(y, Q(y))) ==> (P(∅) \/ Q(∅)) ):
have(thesis) by Prover9
```

SC-TPTP & Friends



Take Home Message

Take Home Message

- You can perform an efficient (tableaux) proof-search while keeping the ability to produce a proof certificate
- You can use `SC-TPTP` to exchange proofs between various tools

What's Next?

- Standalone tool and proof elaboration
- Integration of theories
- Framework for verification of tableaux proofs: `TableauxRocq`[3]
- Addition of tools into the SC-TPTP ecosystem
- The ProoVer competition at FLoC 2026!

[3] Currently under development. Actually, *right now*, depending on what Johann is doing...

Thank you! ☺

https://github.com/GoelandProver/Goeland

https://github.com/SC-TPTP/sc-tptp